

Systems of Linear Equations

A system of equations is a collection of two or more equations containing common variables. When solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

The equations in the system can be linear or non-linear.

The examples in this handout will be linear equations.

Example of systems of linear equations:

$$1) \quad 6x + 7y = -9$$

$$2) \quad -4x - 5y = 5$$

This example has two equations and two unknowns.

In this handout we will show solutions for the follow methods for solving a system of linear equations.

Substitution

Elimination

Graphing

Systems of Linear Equations

The Method of Substitution :

The method of substitution requires five steps, consider the following two equations:

$$\text{Equation (1) } -7x - 2y = -10$$

$$\text{Equation (2) } 6x + y = 15$$

Step 1: Now solve for y in one of the two equations. We will choose equation (2) because it is the easiest equation to solve for y. We now have:

$$y = 15 - 6x$$

Step 2: Substitute this value of y into equation (1).

$$\text{Equation (1) } \quad -7x - 2y = -10 \quad \text{now becomes}$$

$$\text{New Equation (1) } -7x - 2(15 - 6x) = -10$$

Step 3: Solve for x in the New Equation (1).

$$-7x - 2(15 - 6x) = -10$$

$$-7x - 30 + 12x = -10$$

$$5x - 30 = -10$$

$$5x = 20$$

$$x = 4$$

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The Method of Substitution :

Step 4: Substitute the value of $x = 4$ into the original Equation (1).

$$\begin{aligned}\text{Equation (1)} \quad -7x - 2y &= -10 && \text{now becomes} \\ -7(4) - 2y &= -10 && \text{and solve for } y. \\ -28 - 2y &= -10 \\ -2y &= 18 \\ y &= -9\end{aligned}$$

Step 5: Check your answers by substituting the x and y values in each of the original equations. The left and right side of each equation should be equal.

$$\begin{aligned}\text{Equation (1)} \quad -7x - 2y &= -10 && \text{checking } x = 4 \text{ and } y = -9 \\ -7(4) - 2(-9) &= -10 \\ -28 + 18 &= -10 \\ -10 &= -10\end{aligned}$$

$$\begin{aligned}\text{Equation (2)} \quad 6x + y &= 15 && \text{checking } x = 4 \text{ and } y = -9 \\ 6(4) + (-9) &= 15 \\ 24 - 9 &= 15 \\ 15 &= 15\end{aligned}$$

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The Method of Elimination :

The method of elimination requires four steps, consider the following two equations:

$$\text{Equation (1) } x - 2y = 15$$

$$\text{Equation (2) } 2x + 3y = 2$$

In a two variable problem you will rewrite the equations so when they are added together one of the variables will be eliminated, and solve for the remaining variable.

Step 1: Change Equation (1) by multiplying by -2 to obtain a new and equivalent Equation (1).

$$\text{Equation (1) } \quad x - 2y = 15 \quad \text{multiply both sides by -2}$$

$$\text{New Equation (1) } -2x + 4y = -30$$

Step 2: Add New Equation (1) to Equation (2) to obtain Equation (3).

$$\text{New Equation (1) } -2x + 4y = -30$$

$$\text{Equation (2) } \quad 2x + 3y = 2$$

$$\text{Equation (3) } \quad 7y = -28 \quad \text{now we solve for x.}$$

$$y = -4$$

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The Method of Elimination :

Step 3: Substitute the value of $y = -4$ into the original Equation (1).

$$\begin{aligned}\text{Equation (1)} \quad x - 2y &= 15 && \text{now becomes} \\ x - 2(-4) &= 15 && \text{and solve for } x. \\ x + 8 &= 15 \\ x &= 7\end{aligned}$$

Step 4: Check your answers by substituting the x and y values in each of the original equations. The left and right side of each equation should be equal.

$$\begin{aligned}\text{Equation (1)} \quad x - 2y &= 15 && \text{checking } x = 7 \text{ and } y = -4 \\ 7 - 2(-4) &= 15 \\ 7 + 8 &= 15 \\ 15 &= 15\end{aligned}$$

$$\begin{aligned}\text{Equation (2)} \quad 2x + 3y &= 2 && \text{checking } x = 7 \text{ and } y = -4 \\ 2(7) + 3(-4) &= 2 \\ 14 - 12 &= 2 \\ 2 &= 2\end{aligned}$$

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The Graphing Method :

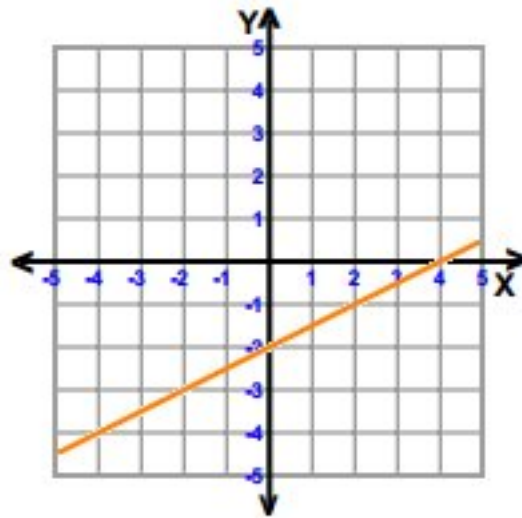
The graphing method requires three steps, consider the following two equations:

$$\text{Equation (1) } y = \frac{1}{2}x - 2$$

$$\text{Equation (2) } y = -2x + 3$$

In a two variable problem you will graph each equation and note the point of intersection.

Step 1: Graph Equation (1), as shown on the graph below.

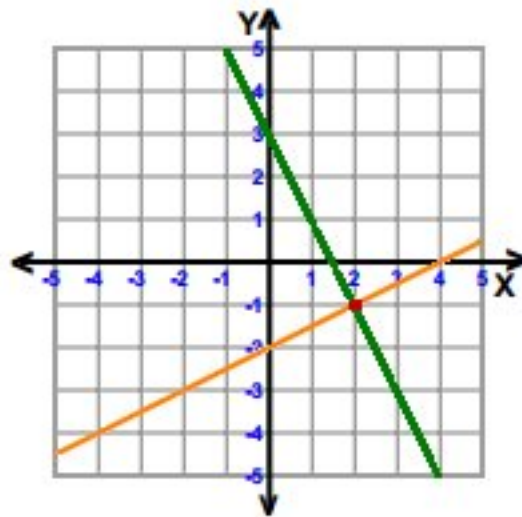


$$\text{Equation (1) } y = \frac{1}{2}x - 2$$

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The Graphing Method :

Step 2: Now Graph Equation (2) on the same graph with Equation (1), as shown on the graph below.



Equation (1) $y = \frac{1}{2}x - 2$

Equation (2) $y = -2x + 3$

Step 3: The solution is the intersection of the two graphs is (2,-1).