Title: Simultaneous Equations.

Target: On completion of this worksheet you should be able to solve two linear simultaneous equations by elimination.

An equation with two unknowns has many solutions.

Example.

$a + 2b = 10$ has solutions:

$a = 1, b = 4.5;$
$a = 2, b = 4;$
$a = 4, b = 3;$

and many more!

If we have two equations with two unknowns then there is often one solution that satisfies both equations.

Example.

$a + 2b = 10$ and $b + a = 6$ both have a solution $a = 2$ and $b = 4$.

Similarly if there are three equations with three unknowns there is often one solution that satisfies all three equations etc.

Finding the solutions that satisfy all the equations given is called **solving the equations simultaneously.**

In order to solve simultaneous equations we need to remember two algebraic tools that we have met previously.

- We can multiply or divide the whole of an equation by an amount.
- We can solve equations with one unknown.

If you have difficulty with this then refer to the algebra sheet on linear equations.

In addition we need to learn a new algebraic tool:

- We can add or subtract a pair of equations to obtain a new equation.

We can see that this makes sense by considering a pence to be the price of an apple and b pence to be the price of a banana.
If someone buys one apple and two bananas for 64p and his friend buys one apple and one banana for 42p then two apples and three bananas must cost 106p. That is

• $1a + 2b = 64$
, $1a + b = 42$

Adding • equations , and gives:

$$(1a + 2b) + (1a + 1b) = 64 + 42.$$

On simplifying we obtain:

$$2a + 3b = 106.$$

Exercise.

If $x + y = 5$ and $3x + 4y = 18$ solve the following:

1 $4x + 5y =$
2 $2x + 3y =$
3 $2x + 2y =$
4 $x + 2y =$
5 $3x + 3y =$
6 $y =$
7 $6x + 8y =$

(Answers: 23, 13, 10, 8, 15, 3, 36.)

When we have a pair of equations with two unknowns we try to add or subtract the equations to get rid of one of the unknowns. This is called the process of **elimination.** This leaves us with an equation with one unknown that we can then solve.

<u>Examples.</u>

1. If $2x + y = 5$ and $x + y = 3$, find x.

•	$2x + y = 5$
,	$x + y = 3$
• − ,	$(2x + y) - (x + y) = 5 - 3$
	$\Rightarrow x = 2.$

2. If $x - y = 7$ and $2x + y = 23$, find x.

•	$x - y = 7$
,	$2x + y = 2$
• + ,	$(x - y) + (2x + y) = 30$
	$\Rightarrow 3x = 30 \quad [\div 3]$
	$\Rightarrow x = 10.$

Notice that the numbers circled at the side name the equations and then explain what we are doing with them in each successive line. This avoids confusion in more complex examples.

If you have difficulty with this then refer to the algebra sheet on brackets.

To find the solution to the equations we must find both unknowns. Once we have found one unknown we can substitute the value of it into one of the original equations to find the other unknown.

<u>Example.</u>

In example 1) above we substitute $x = 2$ into $2x + y = 5$ to obtain

$$4 + y = 5.$$

We solve this equation to obtain

$$y = 1.$$

Therefore the full solution is $x = 2$, $y = 1$.

<u>Exercise.</u>

Solve the following simultaneous equations.

1. $3w + s = 10$, $2w + s = 7$.
2. $4t + r = 12$, $2t + r = 8$.
3. $5p + 2y = 11$, $2y + p = 3$.
4. $6h + l = 19$, $5h - l = 14$.
5. $3j - 2k = 3$, $5j + 2k = 37$.
6. $3c - 4d = 32$, $4d + c = 0$.

(Answers: w=3, s=1; t=2, r=4; p=2, y=1/2; h=3, l=1; j=5, k=6; c=8, d=-2)

So far the equations we have been given have had the same number of one of the unknowns. In general this will not be the case. We therefore need to multiply one or both of the equations by an amount so that we obtain two new equations with the same amount of one unknown.

We should:

- Obtain two equations with the same number of one unknown.
- Eliminate one of the unknowns and solve.
- Substitute the value of this unknown into one of the original equations and solve.

<u>Examples.</u>

1. Solve $2x + 3y = 18$ and $x + 2y = 11$.

•	$2x + 3y = 18$	
,	$x + 2y = 11$	
2× ,	$2x + 4y = 22$	f
• − f	$(2x + 3y) - (2x + 4y) = 18 - 22$	
	$\Rightarrow -y = -4$	
	$\Rightarrow y = 4.$	

Substituting $y = 4$ into • gives

$$2x + 12 = 18.$$

Solving gives $x = 3$.

2. Solve $2x + 3y = 20$ and $3x - 2y = 17$.

•	$2x + 3y = 20$	
,	$3x - 2y = 17$	
3× •	$6x + 9y = 60$	f
2× ,	$6x - 4y = 34$	„
f − „	$13y = 26 \quad [\div 13]$	
	$\Rightarrow y = 2.$	

Substituting $y = 2$ into • gives $2x + 6 = 20$.

Solving gives $x = 7$.

Exercise.

Solve the following pairs of simultaneous equations.

1. $2x - y = 4$, $x + 2y = -2$.
2. $3x - y = 6$, $2x + 3y = 4$.
3. $2x - 3y = 9$, $4x - y = 8$.
4. $2x + 3y = 11$, $4x + y = 12$.
5. $3x + 4y = 25$ $4x - 3y = 0$.
6. $2x + 5y = 16$, $3x - 2y = 5$.
7. $a + 3b = 7$, $a + b = 3$.
8. $3x - 2y = 10$, $x + 2y = 6$.
9. $5a - 3b = 16$, $4a + 2b = 4$.
10. $2a + 5b = 11$, $7a + 3b = -5$.

(Answers: x=1.2, y=-1.6; x=2, y=0; x=1.5, y=-2; x=2.5, y=2; x=3, y=4; x=3,y=2; a=1, b=2; x=4, y=1; a=2, b=-2; a=-2, b=3.)