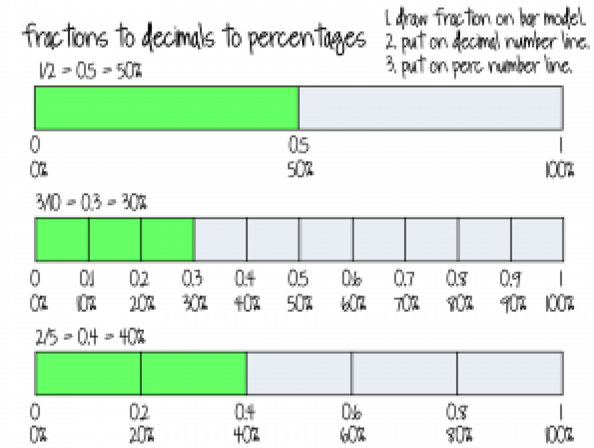
Algebra Tiles- from counting to completing the square

Happy teaching.

I have become increasingly interested in visual models recently as a way of introducing topics. Visual models have the power to illustrate concepts in their rawest, simplest form without the misleading associations that words and abstract notation can introduce. I'm convinced concrete/visual model introductions should form an increased part of my practice, but the question that interests me is ***which*** visual models should I use? There are obviously many considerations, but one is how comprehensively they cover the syllabus. A visual model that demonstrates expanding brackets particularly well would not be that useful if it could not also demonstrate the concept of factorising. Through much reading I believe I have found two visual models that cover the vast majority of number and algebra topics. They are almost mutually exclusive too, complementing each other by covering different, rather than overlapping parts of the curriculum.



The first visual model is ***bar modelling***. The more I experiment with it in lessons, the more I am convinced it can open the door to so many topics that many mid-to-low attaining learners previously found inaccessible. I wrote recently (click here to view) about all the many different topics bar modelling can be used for- from basic fractions work, through FDP, ratio and up to reverse percentages and compound interest.

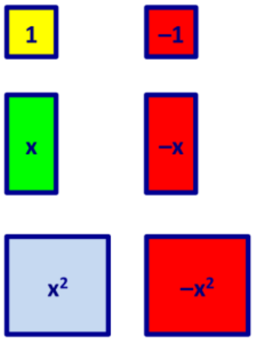
In contrast to bar modelling, I believe ***algebra tiles*** is a very powerful concrete/visual modelling technique that can be used to develop conceptual understanding of topics. In this post I will explain how the algebra tiles model works and demonstrate how it can be used to introduce a great many topics that bar models are not suitable for.

Before I go any further I want to make it clear that I do not think all students need to experience visual models when topics are introduced. Teachers should use them selectively when they think they are needed and will support students' learning. Visual models often fall down for particular variations of question and they are not meant to replace abstract reasoning, merely be a bridge to it for students that need it.

Basic rules of algebra tiles

It is anticipated that you will make these tiles as physical resources for students to use.

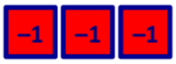
Types of tile



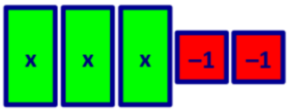
- Six types of tile can represent constants, linear and quadratic terms.
- Each tile has its negative version which is red
- Each physical tile has its positive version on one side and its negative version on the other

Basic representations

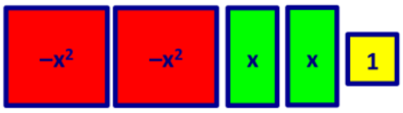
represents +5



represents –3

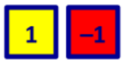


represents $3x - 2$



represents $-2x^2 + 2x + 1$

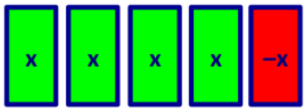
Zero Pairs



‘Zero pairs’ are equal to zero and cancel each other out

This represents +4

This also represents +4 since the two zero pairs are both equal to zero



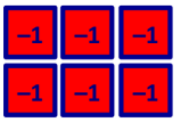
Zero pairs can be formed with the other tiles. This would represent +3x

Four operations using algebra tiles including with negative numbers

Directed Number Addition

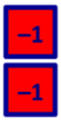
3 + 2

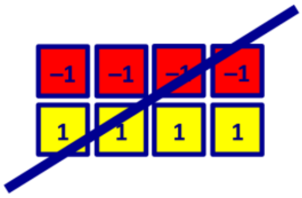
Start with 3 yellow tiles, add 2 more yellow tiles. You now have +5



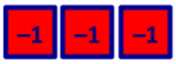
−6 + 4

Start with 6 red tiles, add 4 yellow tiles. Cancel the zero pairs. You now have −2.

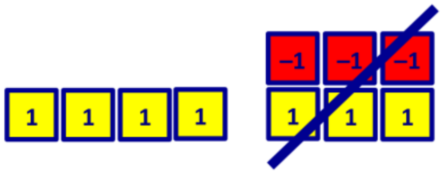


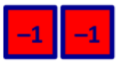


Directed Number Addition

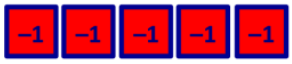


–3 + 7
Start with 3 red tiles,
add 7 yellow tiles.
Cancel the zero pairs.
You now have +4



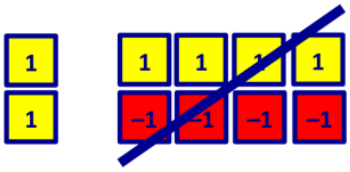


–2 + (–3)
Start with 2 red tiles,
add 3 more red tiles.
You now have –5

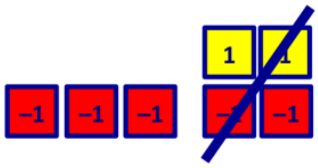


Directed Number Addition

6 + (–4)
Start with 6 yellow
tiles, add 4 red tiles.
Cancel the zero pairs.
You now have +2



2 + (–5)
Start with 2 yellow
tiles, add 5 red tiles.
Cancel the zero pairs.
You now have –3



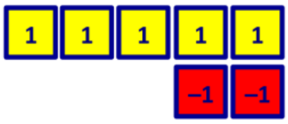
Directed Number Subtraction

5 – 3

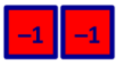
Start with 5 yellow tiles, take away 3 yellow tiles. You now have +2

3 – 5

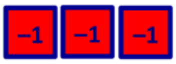
Start with 3 yellow tiles. There aren't 5 yellow tiles to take away, so put in some zero pairs until you have 5 yellows to take away, leaving –2



Take away 5 yellows

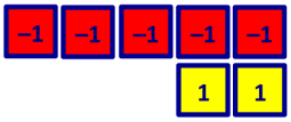


Directed Number Subtraction

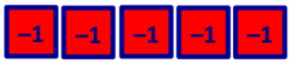


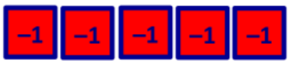
–3 – 2

Start with 3 red tiles. There aren't 2 yellow tiles to take away, so put in some zero pairs until you have 2 yellows to take away, leaving –5



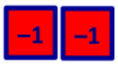
Take away 2 yellows



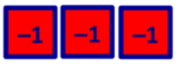


–5 – (–3)

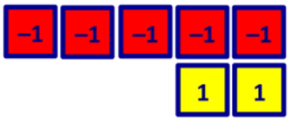
Start with 5 red tiles, take away 3 red tiles. You now have –2



Directed Number Subtraction

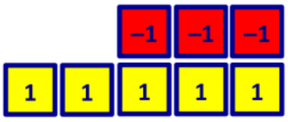


–3 – (–5)
Start with 3 red tiles. There aren't 5 red tiles to take away so put in zero pairs until you can take away 5 red tiles. You now have +2



Take away 5 reds

2 – (–3)
Start with 2 yellow tiles. There aren't 3 red tiles to take away so put in zero pairs until you can take away 3 red tiles. You now have +5

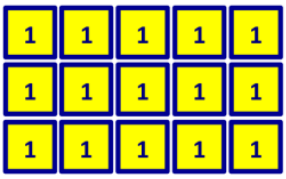


Take away 3 reds

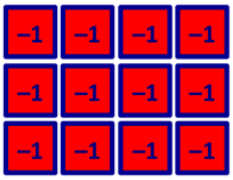
Directed Number Multiplication

The first number in a multiplication is how many rows you have. The second number is how many tiles in each row.

3 X 5 = 15
3 rows of 5 yellows



3 X –4 = –12
3 rows of 4 reds

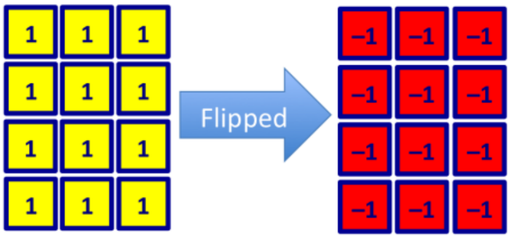


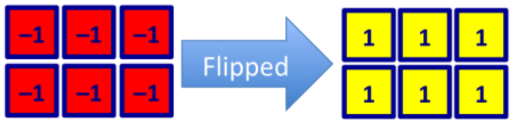
Directed Number Multiplication

If the first number is negative, you do that (absolute value) number of rows and then you flip them.

–4 X 3 = –12
4 rows of 3 yellows-
then flipped

–2 X –3 = 6
2 rows of 3 reds-
then flipped



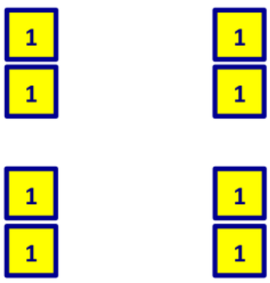


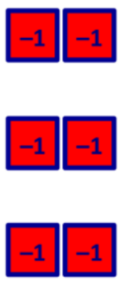
Directed Number Division

The first number in a division is how many tiles you have. The second number is how many groups you form. The answer is how many tiles in each group.

8 ÷ 4 = 2
4 groups, +2 in each

–6 ÷ 3 = –2
3 groups, –2 in each

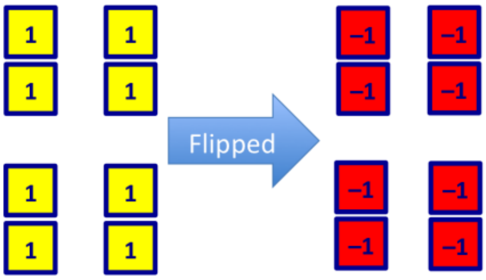




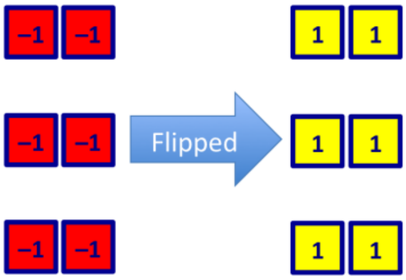
Directed Number Division

The second number is negative, make that (absolute value) number of groups and then you flip them.

8 ÷ –4 = –2
4 groups, flipped, –2 in each



–6 ÷ –3 = 2
3 groups, flipped, +2 in each



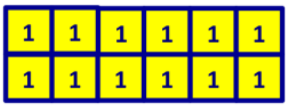
Types of number

Factors and primes

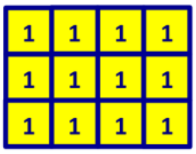
To find factors of a number, organise the tiles into rectangles and record the numbers of rows and columns.

Find the factors of 12

1 row of 12



2 rows of 6



3 rows of 4

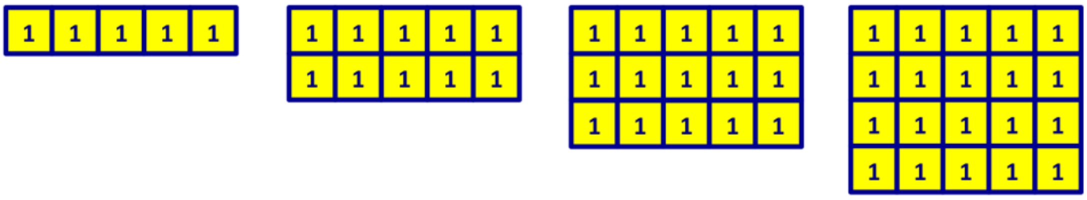
Factors of 12 are 1, 2, 3, 4 ,6, 12

The primes are the numbers that can only form 1 rectangular arrangement.

Multiples

To find multiples of a number you create a row of that many tiles and then keep adding on more rows.

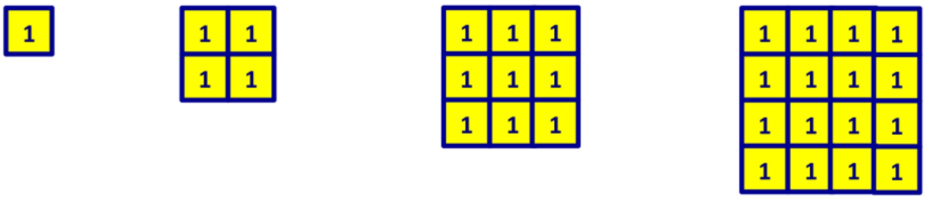
List the multiples of 5



5　　10　　15　　20

Squares

Square numbers come from arranging the tiles into squares of increasing size



1　　4　　9　　16

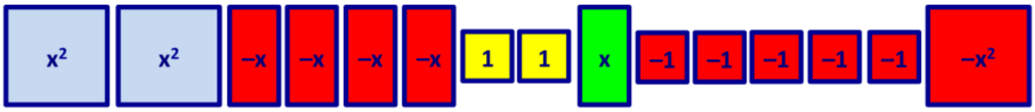
All of the above concepts are relatively trivial and you may have used variations on the algebra tile model when teaching them. For example, using multi-link cubes to derive factors or show the square numbers. However, what is powerful about the algebra tiles model is how seamlessly is transfers to algebraic concepts…

Collecting like terms

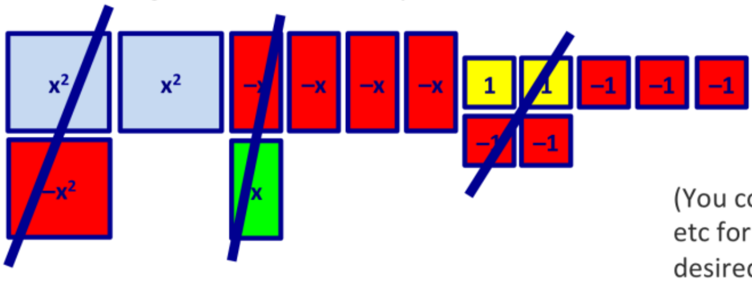
Collecting like terms

Represent your expression using algebra tiles, rearrange to form zero pairs which cancel out leaving your simplified expression.

Simplify $2x^2 – 4x + 2 + x – 5 – x^2$



Rearrange to form zero pairs. Cancel out. Leaving $x^2 – 3x – 3$



(You could also create a, b, c tiles etc for different letter terms if desired for this objective)

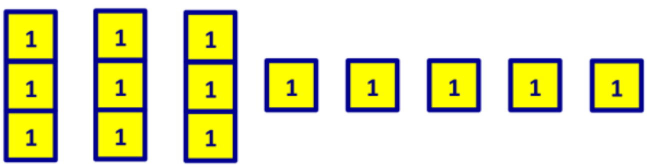
Substitution

Substitution

Only really useful as a very basic introduction to the topic, but does get across the concept nicely. Form your expression. Swap x terms with +1 or –1 tiles once you know the value of x. Cancel out any zero pairs

If $x = 3$, fine the value of $3x + 5$

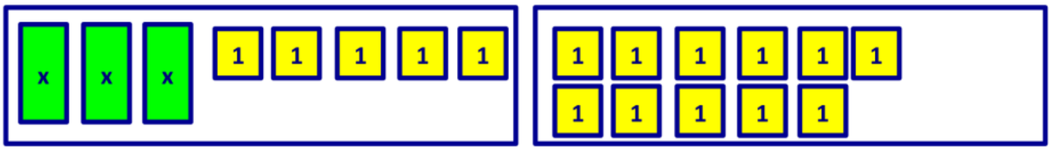
Substitute 3 yellows for every x. $3x + 5 = 14$



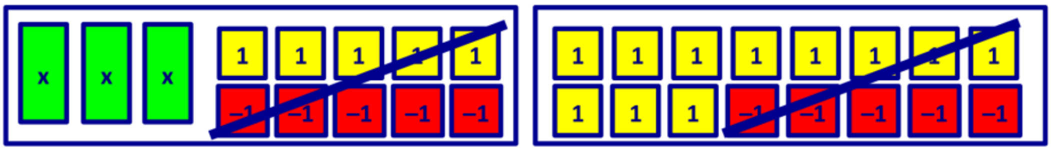
Solving equations

Use a balancing approach with inverse tiles to eliminate others, working your way to having just x tiles left on one side.

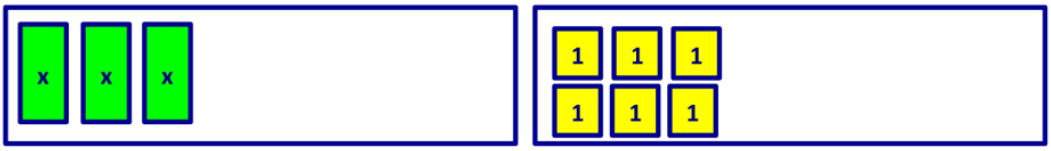
Solve 3x + 5 = 11



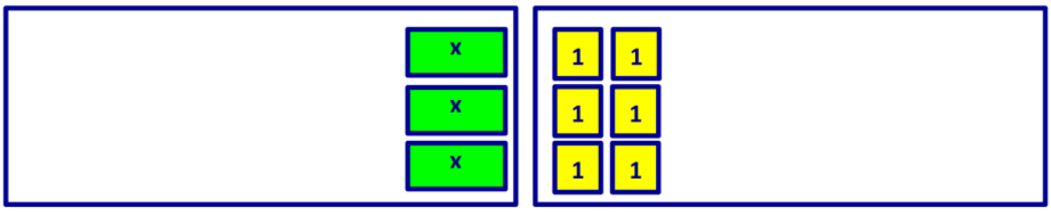
Eliminate the 5 yellows on the left with 5 reds (forming zero pairs). Do the same to both sides.



Solving equations



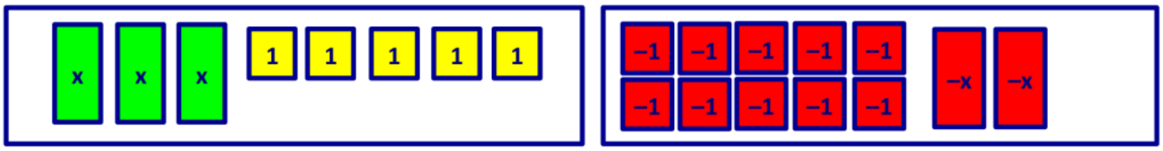
Then group the yellow tiles equally against the x tiles



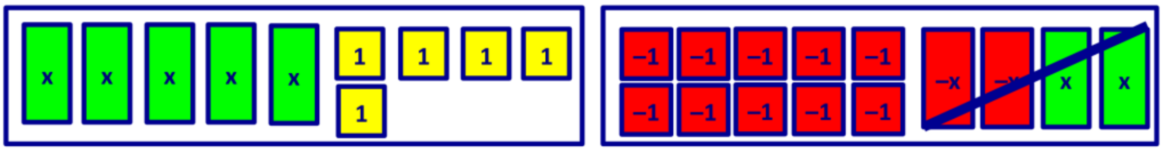
x = 2

Solving equations

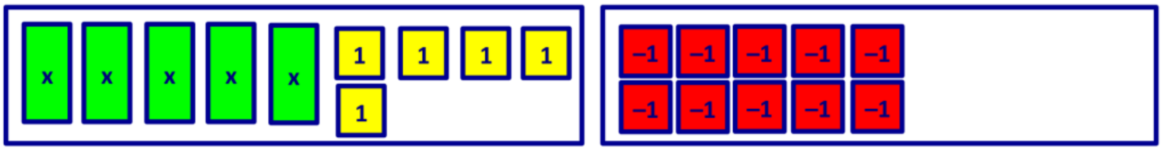
Solve 3x + 5 = –10 – 2x



Eliminate the 2 red x tiles by adding 2 green x tiles to both sides.

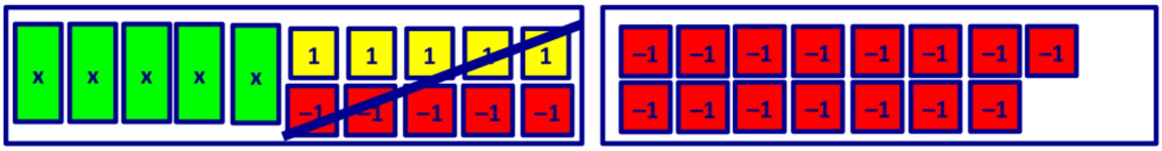


This leaves 5x + 5 = –10

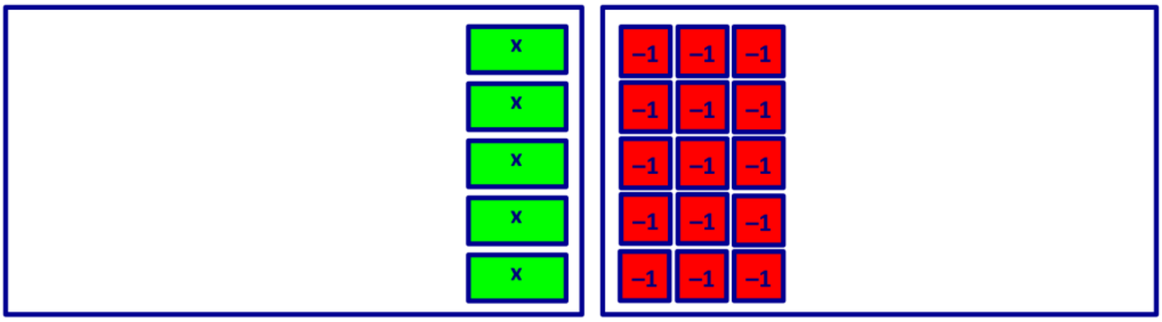


Solving equations

Next eliminate the yellow tiles on the left by adding –5 to both sides.



This leaves 5x = –15. Group the -1 tiles equally against the x tiles.

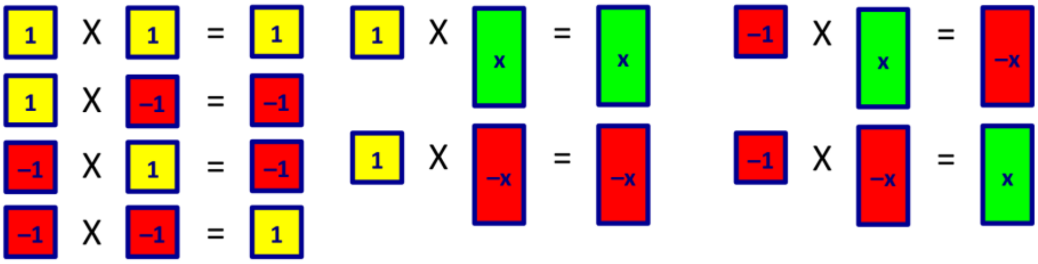


x = –3

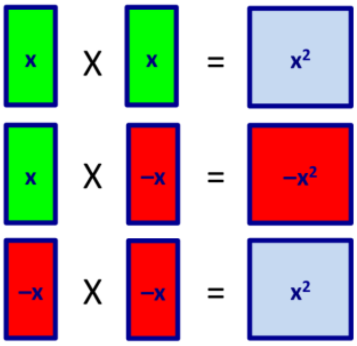
Expanding brackets

Expanding

Students need to know/revise basic multiplications such as:

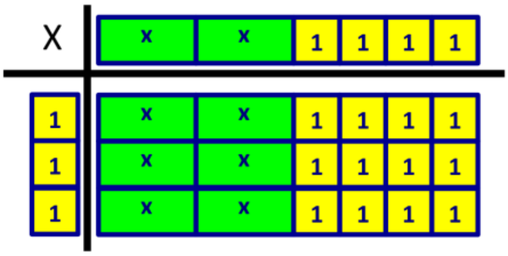


You then use a multiplication grid format to expand brackets.



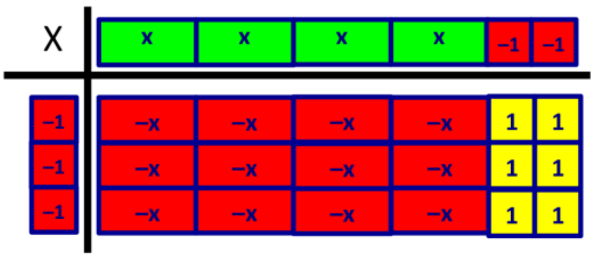
Expanding single brackets

Expand 3(2x + 4)



6x + 12

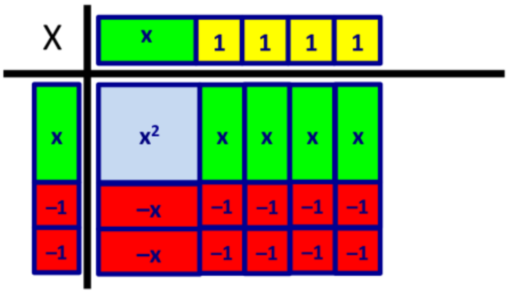
Expand −3(4x – 2)



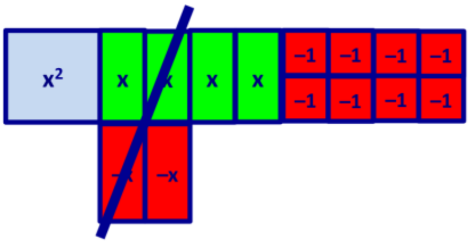
−12x + 6

Expanding double brackets

Expand (x + 4)(x – 2)



Rearrange and cancel out the zero pairs of x terms.



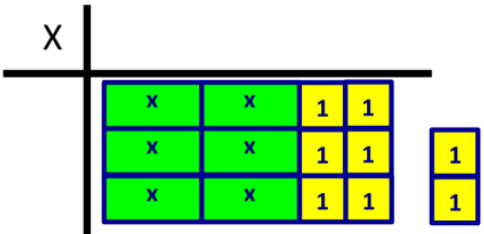
$x^2 + 2x – 8$

Factorising

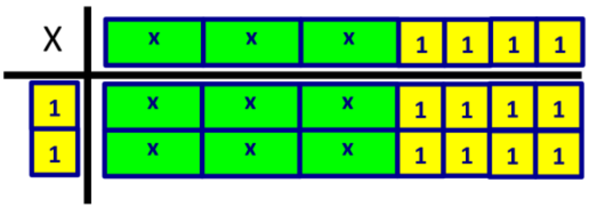
Factorising- HCF

You treat it a bit like a jigsaw puzzle. You put your expression inside the multiplication grid and have to rearrange it until it forms a rectangle (with no remainders). You then think about what was multiplied to make each term.

Factorise 6x + 8
Fit the tiles into the grid to form a rectangle



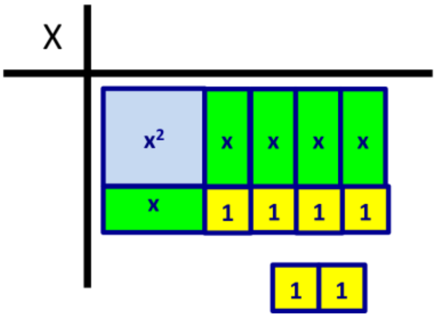
Can't get the 1s to fit- not a solution



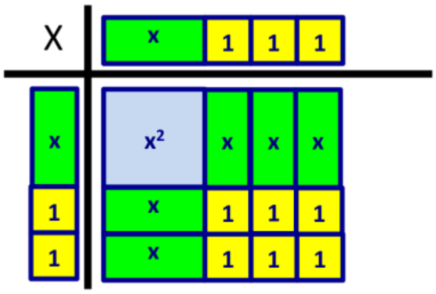
The tiles form a rectangle.
Now think about what needs to be multiplied to make each tile…
2(3x + 4)

Factorising- double brackets

Factorise $x^2 + 5x + 6$



2 spare tiles- no solution

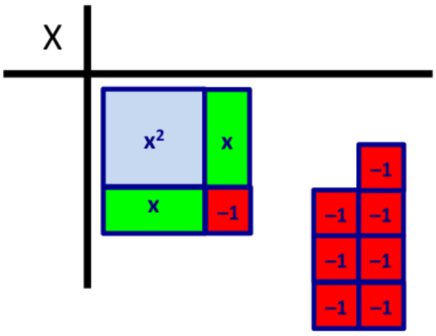


Rectangle formed with no remainders. Think about what was multiplied to make each tile…

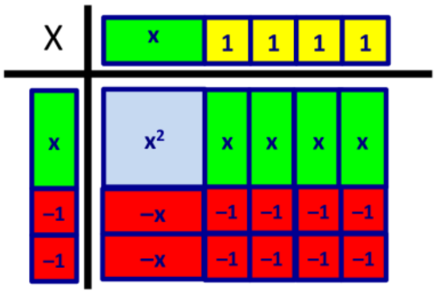
$(x + 3)(x + 2)$

Factorising- double brackets

Factorise $x^2 + 2x – 8$



Spare –1 tiles and not another arrangement possible to form a rectangle. We'll put in some zero pairs of x terms until can get it to work.

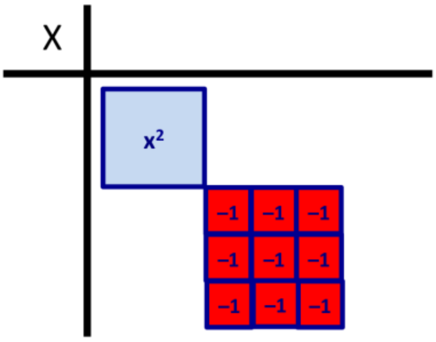


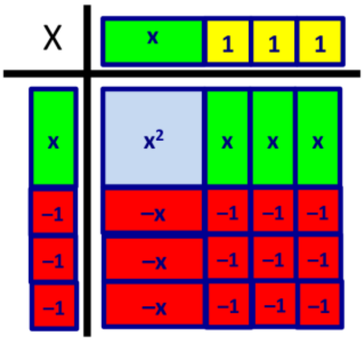
Found a possible solution after adding 2 zero pairs of x terms…

$(x + 4)(x – 2)$

Factorising- difference of two squares

Factorise $x^2 - 9$





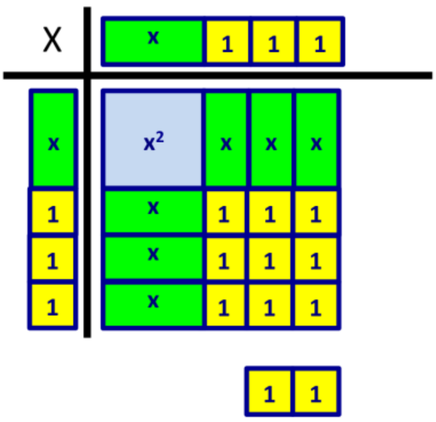
No x terms so put in zero pairs of x terms to get it to work.

$(x + 3)(x - 3)$

Completing the square

Completing the square

Write $x^2 + 6x + 11$ in the form $(x + a)^2 + b$



Both factors are the same so you need to share your x terms equally between columns and rows.

The remainder 1s left over is the b term.

$(x + 3)^2 + 2$