

# The Hyperbola

- ◆ Graph a hyperbola and identify its important parts.
- ◆ Write the equation of a hyperbola in standard form given its important parts.
- ◆ Identify a hyperbola and write the equation in standard form given a conic equation in non standard form.

# Standard Forms of a Hyperbola

◆ The standard form of the equation of an hyperbola with center at  $(h, k)$ .

*Horizontal*

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Vertices  $(h \pm a, k)$

Asymptotes  $y = \pm \frac{b}{a} x$

$$\text{Foci: } c^2 = a^2 + b^2$$

Foci  $(h \pm c, k)$

*Vertical*

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Vertices  $(h, k \pm a)$

Asymptotes  $y = \pm \frac{a}{b} x$

Foci  $(h, k \pm c)$

# #1 Graph the hyperbola

$$9x^2 - 16y^2 = 144$$

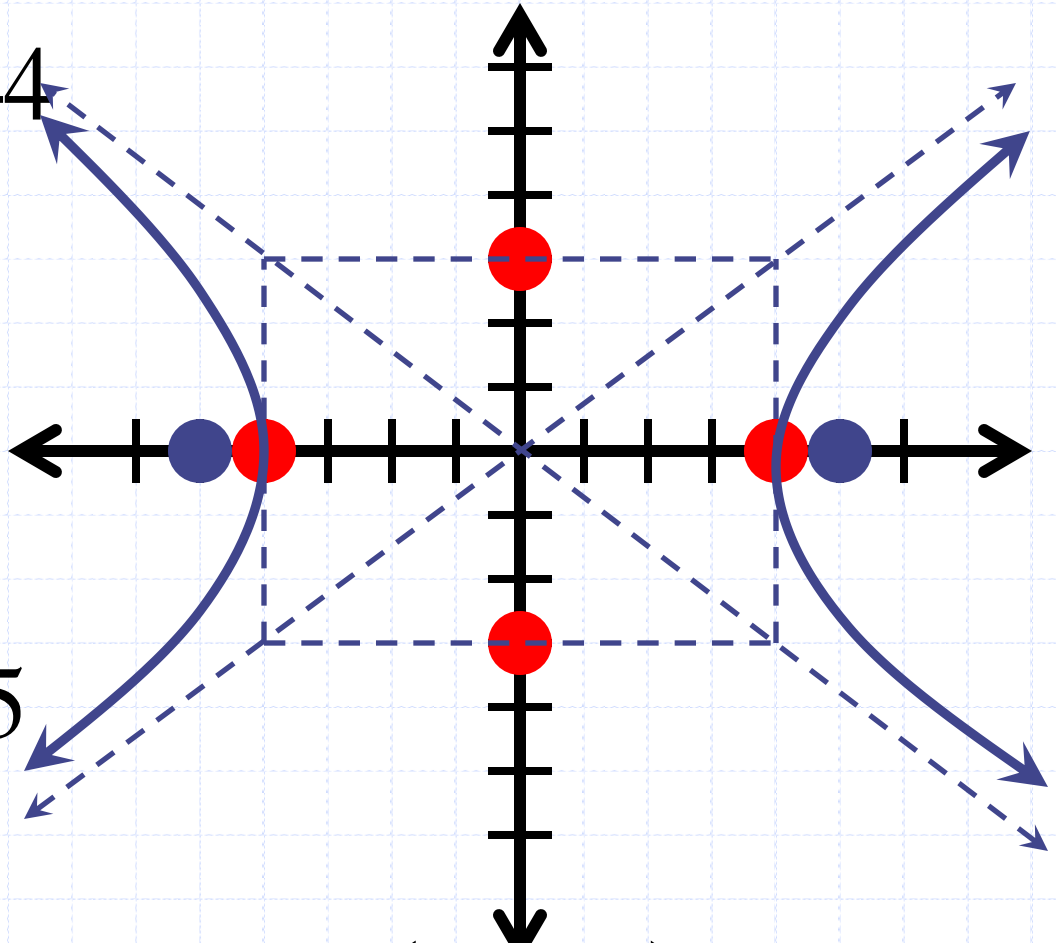
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$a = 4 \quad b = 3$$

$$c = \sqrt{16 + 9} = 5$$

Vertices  $(\pm 4, 0)$

Asymptotes  $y = \pm \frac{3}{4}x$  Focus  $(\pm 5, 0)$



## #2 Graph the hyperbola

$$y^2 - 25x^2 = 25$$

$$\frac{y^2}{25} - x^2 = 1$$

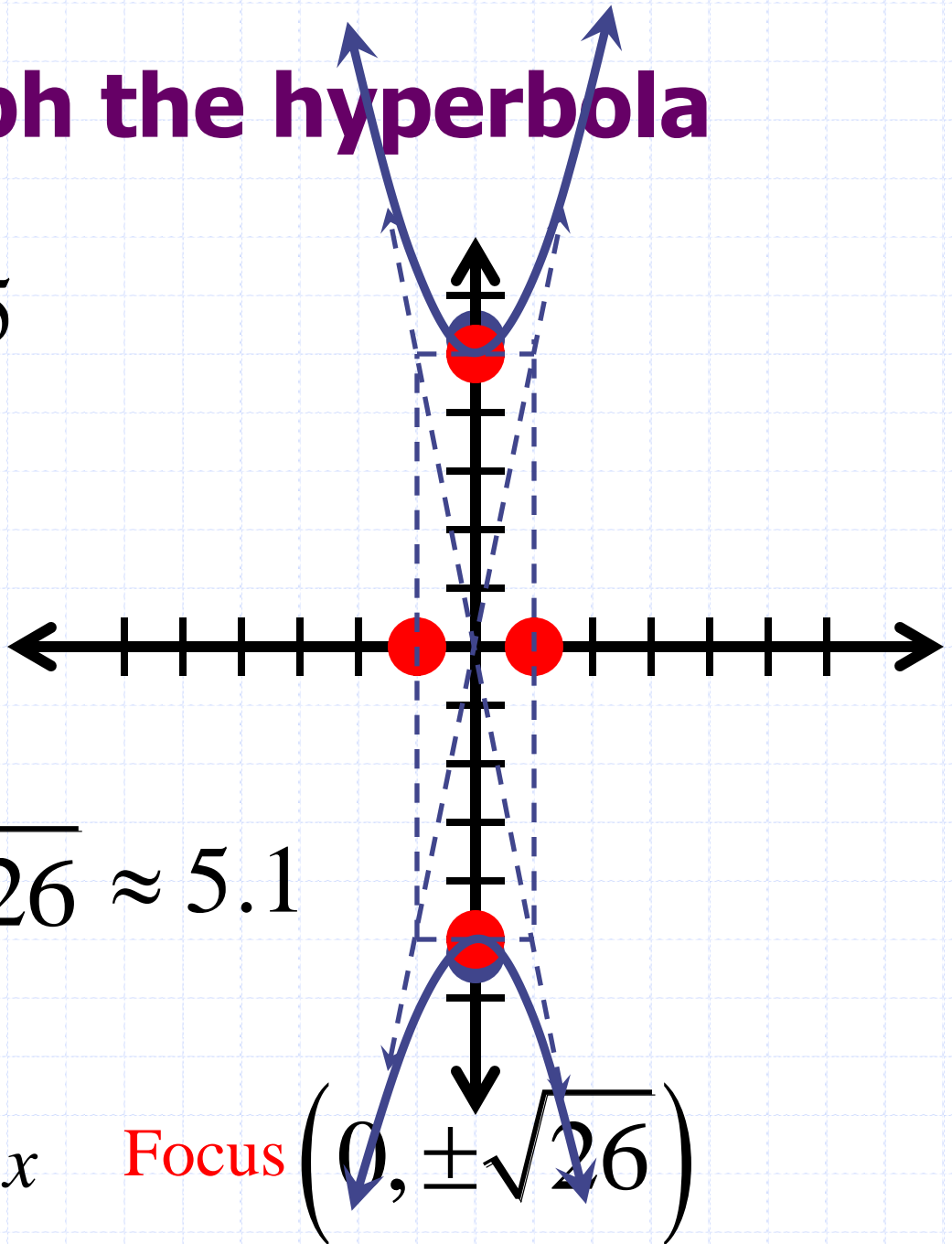
$$a = 5 \quad b = 1$$

$$c = \sqrt{25 + 1} = \sqrt{26} \approx 5.1$$

Vertices  $(0, \pm 5)$

Asymptotes  $y = \pm \frac{5}{1}x$

Focus  $(0, \pm \sqrt{26})$



# #3 Graph the hyperbola

$$\frac{(x-2)^2}{4} - \frac{(y+2)^2}{16} = 1$$

Center  $(2, -2)$

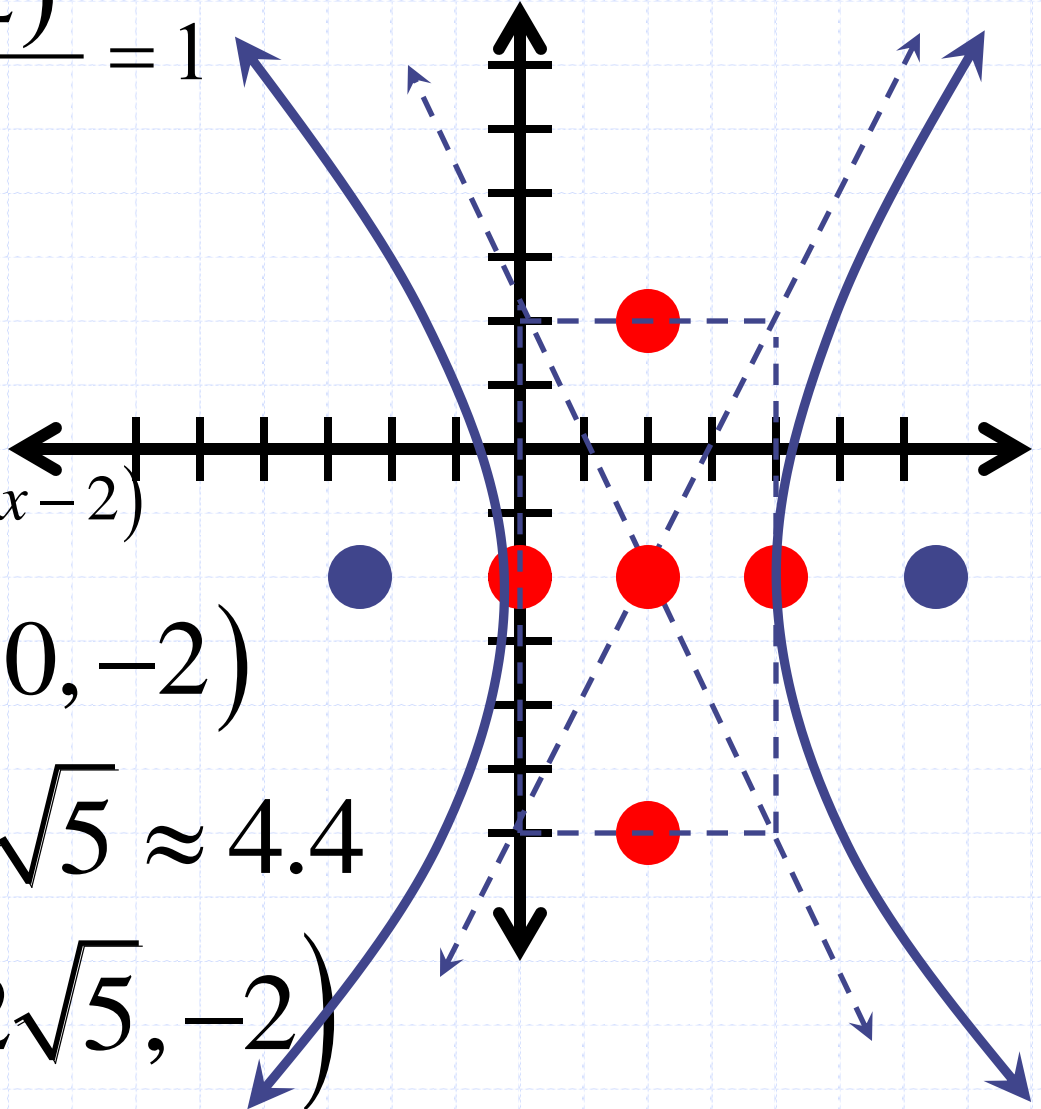
$$a = 2 \quad b = 4$$

Asymptotes  $(y+2) = \pm 2(x-2)$

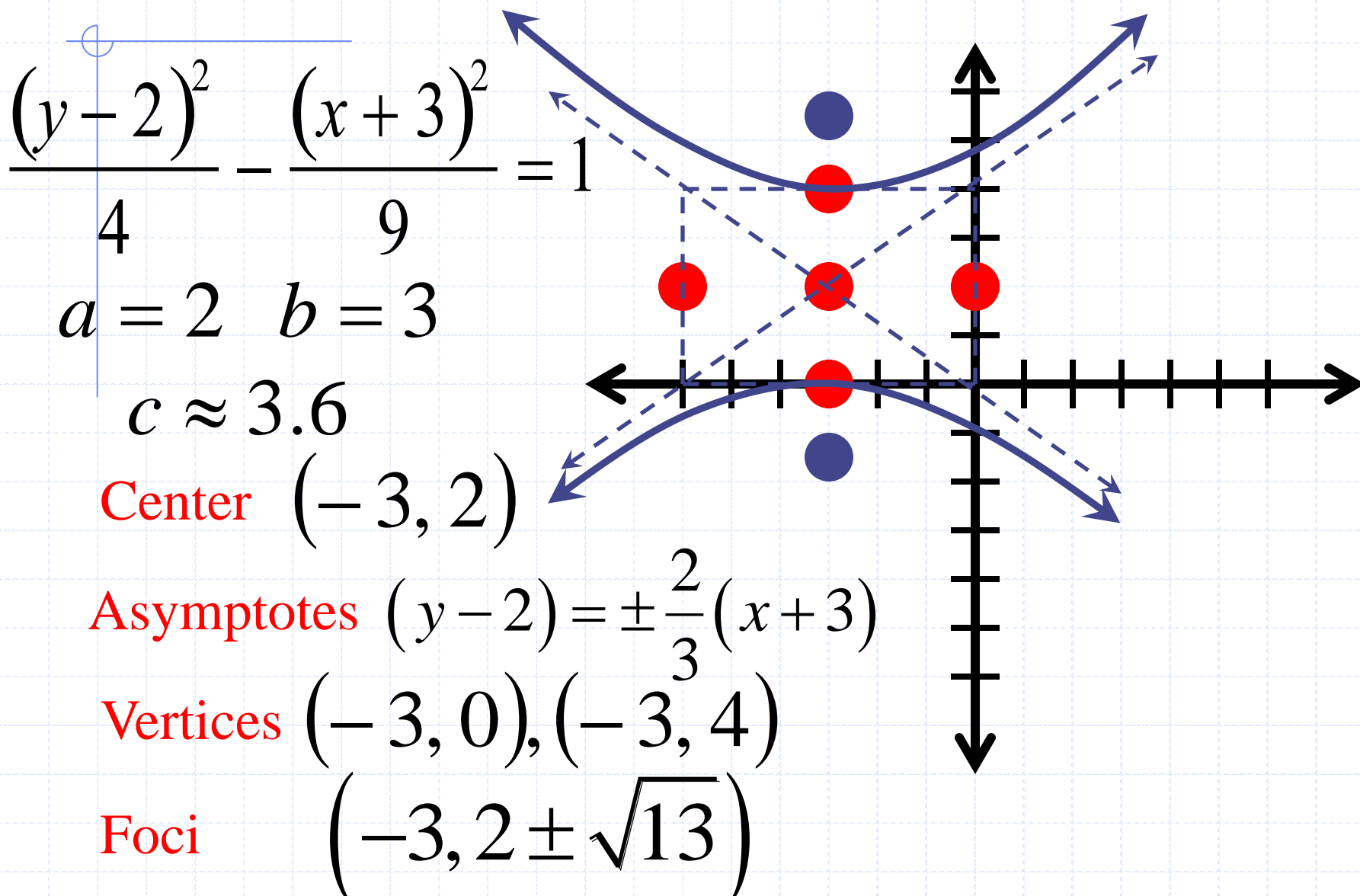
Vertices  $(4, -2), (0, -2)$

$$c = \sqrt{16 + 4} = 2\sqrt{5} \approx 4.4$$

Focus  $(2 \pm 2\sqrt{5}, -2)$



## #4 Graph the hyperbola



# #5 Writing Equations of Hyperbolas

- ◆ Write an equation of the hyperbola with foci at  $(0, \pm 5)$  and vertices at  $(0, \pm 3)$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

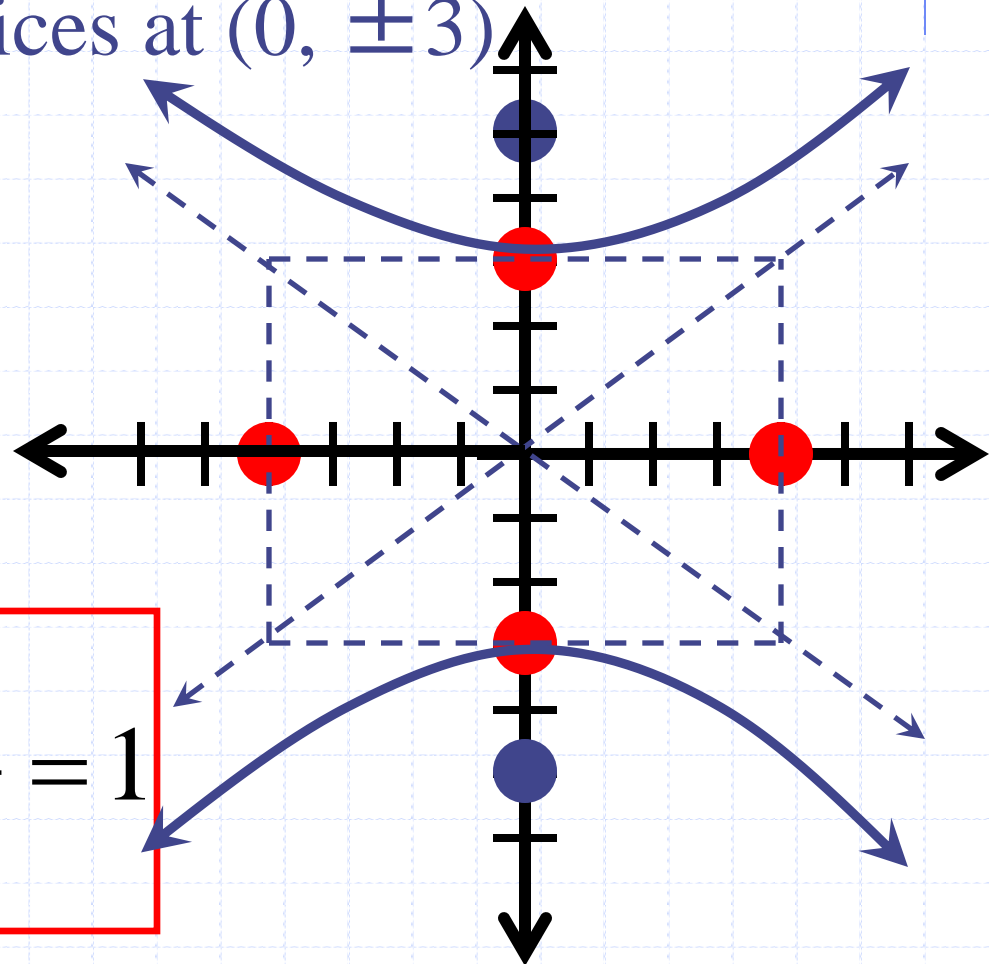
$$a = 3 \quad c = 5$$

$$c^2 = a^2 + b^2$$

$$25 = 9 + b^2$$

$$b = 4$$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$



# #6 Writing Equations of Hyperbolas

◆ Write an equation of the hyperbola with foci at  $(3 \pm \sqrt{10}, 3)$  and vertices at  $(4, 3)$  and  $(2, 3)$ .

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center:  $(3, 3)$

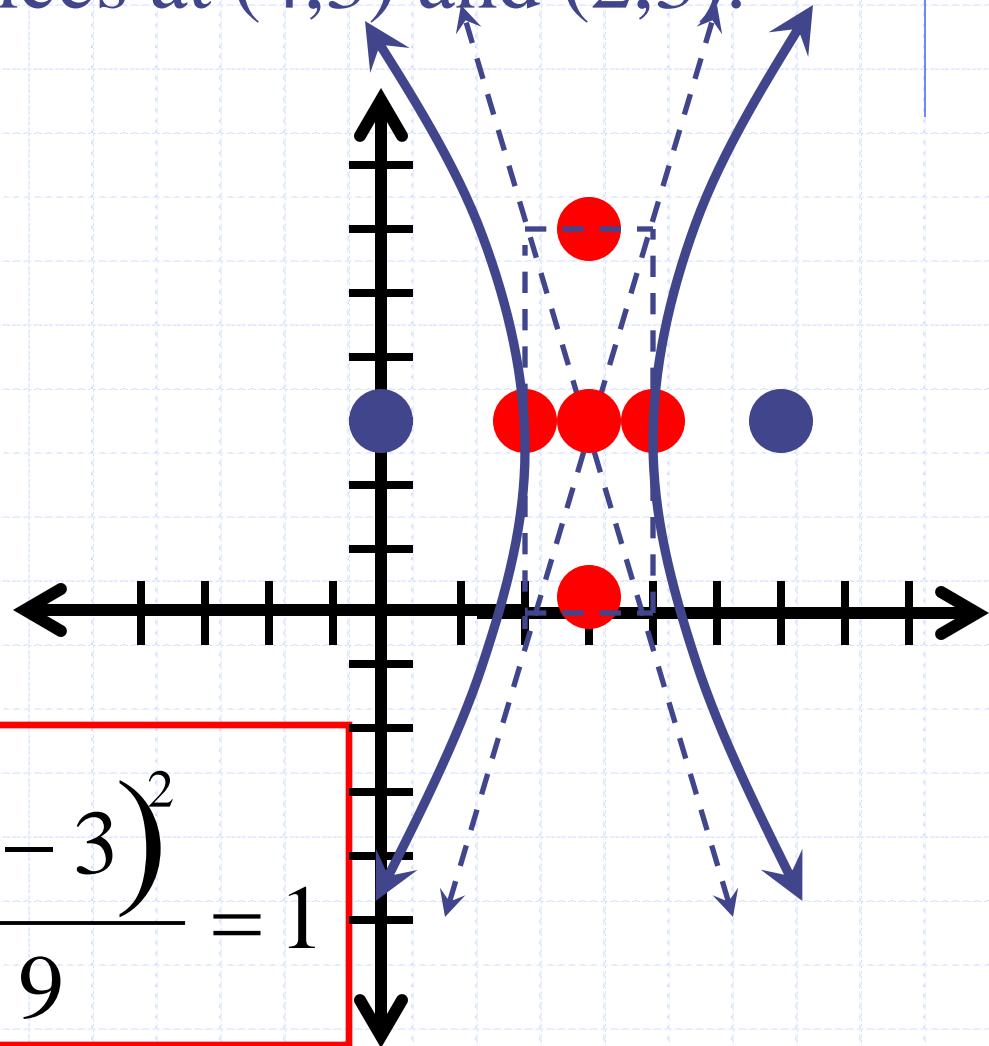
$$a = 1 \quad c = \sqrt{10}$$

$$c^2 = a^2 + b^2$$

$$10 = 1 + b^2$$

$$b = 3$$

$$\frac{(x-3)^2}{1} - \frac{(y-3)^2}{9} = 1$$





# #7 Writing Equations of Hyperbolas

- ◆ Write an equation of the hyperbola with foci at (4,5) and (4,-3) and vertices at (4,4) and (4,-2).

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

Center: (4,1)

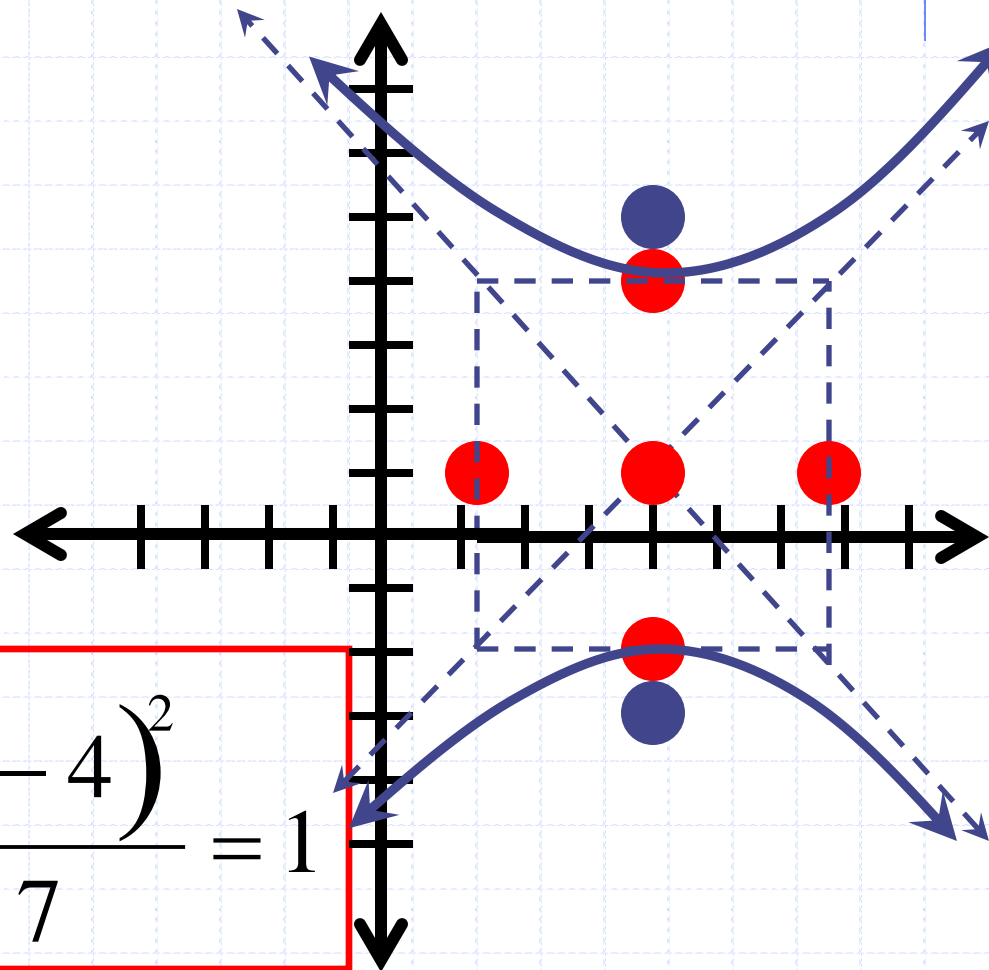
$$a = 3 \quad c = 4$$

$$c^2 = a^2 + b^2$$

$$16 = 9 + b^2$$

$$b = \sqrt{7}$$

$$\frac{(y - 1)^2}{9} - \frac{(x - 4)^2}{7} = 1$$



## #8 Write in Standard Form

$$4x^2 - y^2 - 16x - 4y - 4 = 0$$

$$4x^2 - 16x - y^2 - 4y = 4$$

$$4(x^2 - 4x + \underline{4}) - (y^2 + 4y + \underline{4}) = 4 + 4(\underline{4}) - 1(\underline{4})$$

$$4(x - 2)^2 - (y + 2)^2 = 4 + 16 - 4$$

$$4(x - 2)^2 - (y + 2)^2 = 16$$

$$\frac{(x - 2)^2}{4} - \frac{(y + 2)^2}{16} = 1$$

## #9 Write in Standard Form

$$-9x^2 + 16y^2 + 54x + 64y - 161 = 0$$

$$16y^2 + 64y - 9x^2 + 54x = 161$$

$$16(y^2 + 4y + \underline{4}) - 9(x^2 - 6x + \underline{9}) = 161 + 16(\underline{4}) - 9(\underline{9})$$

$$16(y + 2)^2 - 9(x - 3)^2 = 161 + 64 - 81$$

$$16(y + 2)^2 - 9(x - 3)^2 = 144$$

$$\frac{(y + 2)^2}{9} - \frac{(x - 3)^2}{16} = 1$$