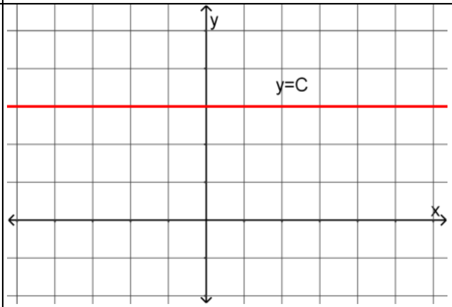
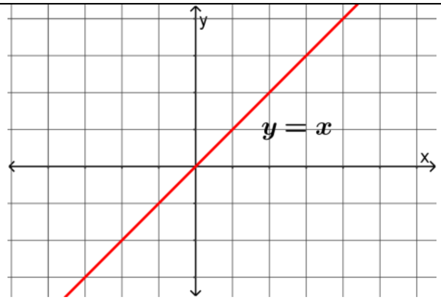
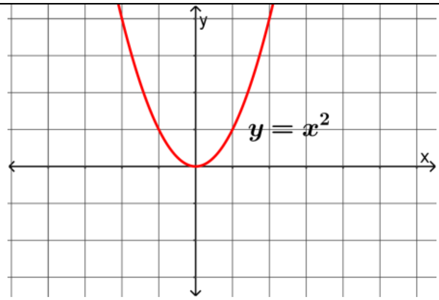
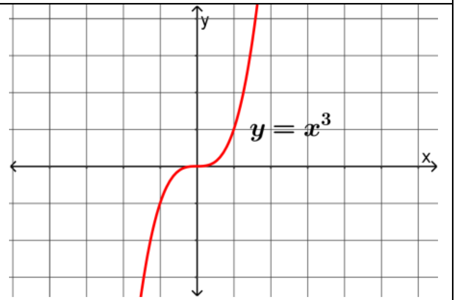
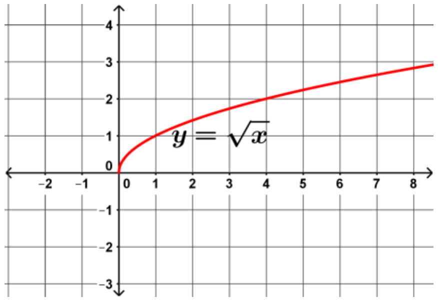
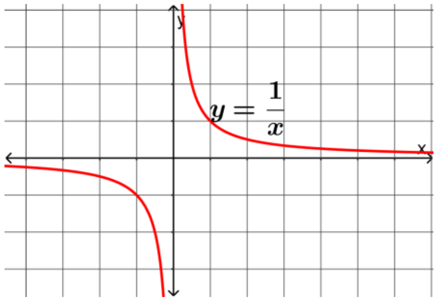
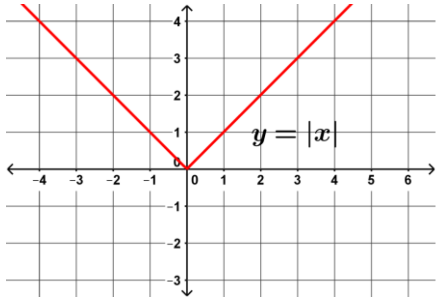
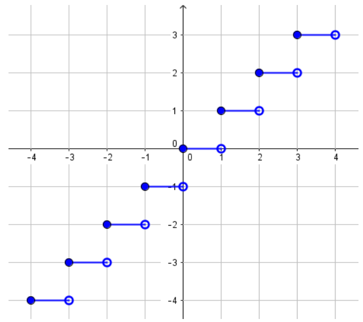
# وصف خصائص الدوال الرئيسية

| الدالة | الدالة الثابتة<br>$y = c$ حيث c عدد حقيقي | الدالة المحايدة (المتطابقة)<br>$y = x$ | الدالة التربيعية<br>$y = x^2$ | الدالة التكعيبية<br>$y = x^3$ |
|---|---|---|---|---|
| الرسم |  |  |  |  |
| المجال | D=ℜ | D=ℜ | D=ℜ | D=ℜ |
| المدى | R=C | R=ℜ | R= $\{y \mid y \geq 0, y \in R\}$ | R=ℜ |
| نقاط التقاطع مع x | x-int= $\phi$ | x-int={0} | x-int={0} | x-int={0} |
| نقاط التقاطع مع y | y-int= $c$ | y-int={0} | y-int={0} | y-int={0} |
| التناظر<br>(زوجية-فردية) | متناظرة حول y<br>دالة زوجية | متناظرة حول نقطة الأصل<br>دالة فردية | متناظرة حول y<br>دالة زوجية | متناظرة حول نقطة الأصل<br>دالة فردية |
| الاتصال | متصلة | متصلة | متصلة | متصلة |
| السلوك الطرفي | | | | |
| فترات التزايد والتناقص | | | | |

| الدالة | الدالة الجذرية $y=\sqrt{x}$ | الدالة المعكوسة (نسبية/كسرية) $y=\frac{1}{x}$ | دالة القيمة المطلقة $y=\lvert x\rvert$ | دالة أكبر عدد صحيح $y=[x]$ |
|---|---|---|---|---|
| الرسم |  |  |  |  |
| المجال | D= $\{x \mid x\geq 0, x\in R\}$ | D= $R\setminus\{0\}=\{x \mid x\neq 0, x\in R\}$ | D=ℝ | D=ℝ |
| المدى | R= $\{y \mid y\geq 0, y\in R\}$ | R= $R\setminus\{0\}=\{y \mid y\neq 0, y\in R\}$ | R= $\{y \mid y\geq 0, y\in R\}$ | R=Z<br>حيث Z الأعداد الصحيحة |
| نقاط التقاطع مع x | x-int=0 | x-int= $\phi$ | x-int={0} | x-int= $\{x \mid 0\leq x<1\}$ |
| نقاط التقاطع مع y | y-int=0 | y-int= $\phi$ | y-int={0} | y-int={0} |
| التناظر (زوجية-فردية) | ليست متناظرة<br>لا دالة زوجية ولا فردية | متناظرة حول نقطة الأصل<br>دالة فردية | متناظرة حول y<br>دالة زوجية | ليست متناظرة |
| الاتصال | متصلة على مجالها | غير متصلة عند الصفر | متصلة | ليست متصلة |
| السلوك الطرفي | | | | |
| فترات التزايد والتناقص | | | | |

# Transformations of Functions

The transformations you have seen in the past can also be used to move and resize graphs of functions. We will be examining the following changes to $f(x)$:

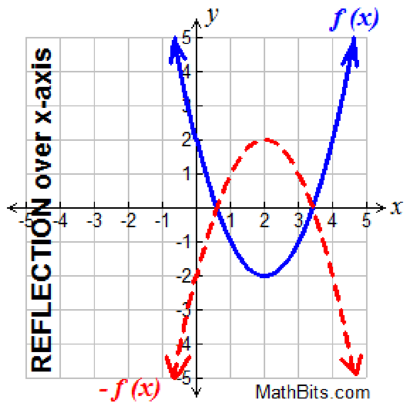
$-f(x)$, $f(-x)$, $f(x)+k$, $f(x+k)$, $kf(x)$, $f(kx)$

reflections | translations | dilations

## Reflections of Functions: $-f(x)$ and $f(-x)$

### Reflection over the $x$-axis.

**$-f(x)$ reflects $f(x)$ over the $x$-axis**

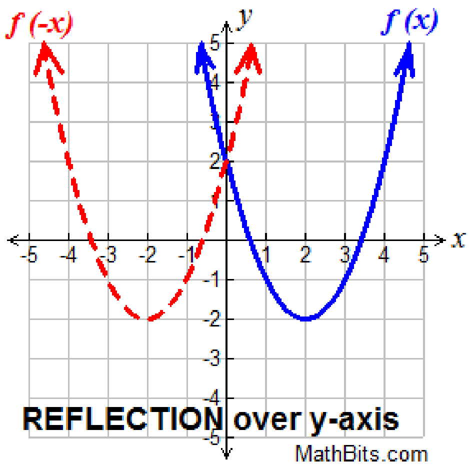



Reflections are mirror images. Think of "folding" the graph over the $x$-axis.

On a grid, you used the formula $(x,y) \rightarrow (x,-y)$ for a reflection in the $x$-axis, where the $y$-values were negated. Keeping in mind that $y = f(x)$, we can write this formula as $(x, f(x)) \rightarrow (x, -f(x))$.

### Reflection over the $y$-axis.

**$f(-x)$ reflects $f(x)$ over the $y$-axis**




Reflections are mirror images. Think of "folding" the graph over the $y$-axis.
On a grid, you used the formula $(x,y) \rightarrow (-x,y)$ for a reflection in the $y$-axis, where the $x$-values were negated. Keeping in mind that $y = f(x)$, we can write this formula as $(x, f(x)) \rightarrow (-x, f(-x))$.

## Translations of Functions: $f\,(x) + k$ and $f\,(x + k)$

### Translation vertically (upward or downward)

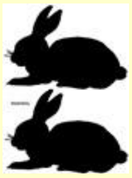
$f\,(x) + k$ translates $f\,(x)$ up or down

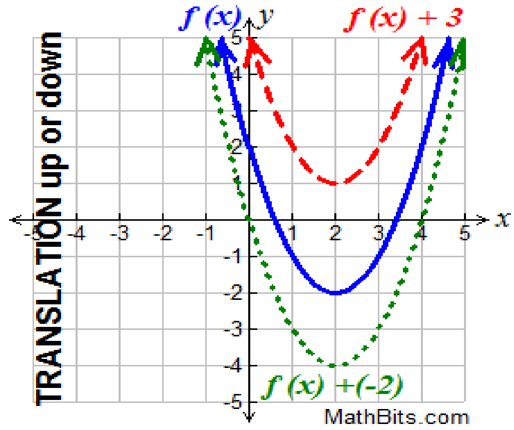
This translation is a "slide" straight up or down.

- if $k > 0$, the graph translates upward $k$ units.
- if $k < 0$, the graph translates downward $k$ units.

On a grid, you used the formula $(x,y) \rightarrow (x,y + k)$ to move a figure upward or downward. Keeping in mind that $y = f\,(x)$, we can write this formula as $(x, f\,(x)) \rightarrow (x, f\,(x) + k)$.

Remember, you are adding the value of $k$ to the $y$-values of the function.






### Translation horizontally (left or right)

$f\,(x + k)$ translates $f\,(x)$ left or right

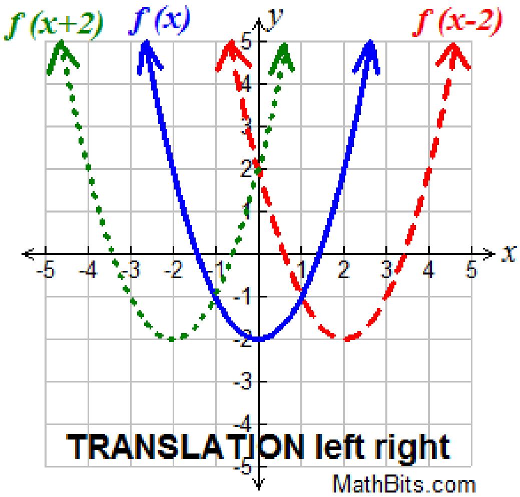
This translation is a "slide" left or right.

- if $k > 0$, the graph translates to the **left** $k$ units.
- if $k < 0$, the graph translates to the **right** $k$ units.

This one will not be obvious from the patterns you previously used when translating points.

A horizontal shift means that every point $(x,y)$ on the graph of $f\,(x)$ is transformed to $(x - k, y)$ or $(x + k, y)$ on the graphs of $y = f\,(x + k)$ or $y = f\,(x - k)$ respectively.

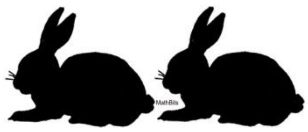
*Look carefully as this can be very confusing!*




*Hint:* To remember which way to move the graph, set $(x + k) = 0$. The solution will tell you in which direction to move and by how much.

$f\,(x - 2)$: $x - 2 = 0$ gives $x = +2$, move right 2 units.

$f\,(x + 3)$: $x + 3 = 0$ gives $x = -3$, move left 3 units.



# Dilations of Functions: $kf(x)$ and $f(kx)$

## Vertical Stretch or Compression (Shrink)

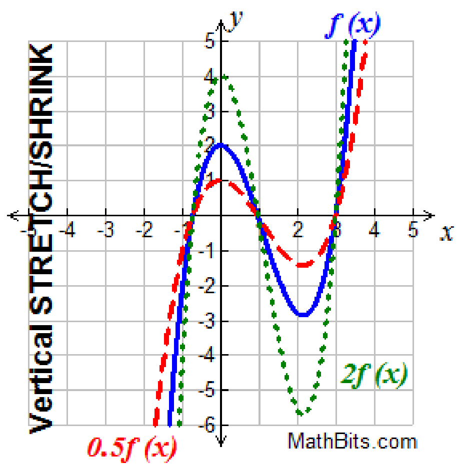
$k\,f(x)$ stretches/shrinks $f(x)$ vertically

A **vertical stretching** is the stretching of the graph away from the $x$-axis

A **vertical compression** (or **shrinking**) is the squeezing of the graph toward the $x$-axis.

- if $k > 1$, the graph of $y = k \cdot f(x)$ is the graph of $f(x)$ **vertically stretched** by multiplying each of its $y$-coordinates by $k$.
- if $0 < k < 1$ (*a fraction*), the graph is $f(x)$ **vertically shrunk (or compressed)** by multiplying each of its $x$-coordinates by $k$.
- if $k$ should be negative, the vertical stretch or shrink is followed by a reflection across the $x$-axis.

Notice that the "roots" on the graph stay in their same positions on the $x$-axis. The graph gets "taffy pulled" up and down from the locking root positions. The $y$-values change.




"Multiply y-coordinates"
(x, y) becomes (x, ky)
"vertical dilation"

## Horizontal Stretch or Compression (Shrink)

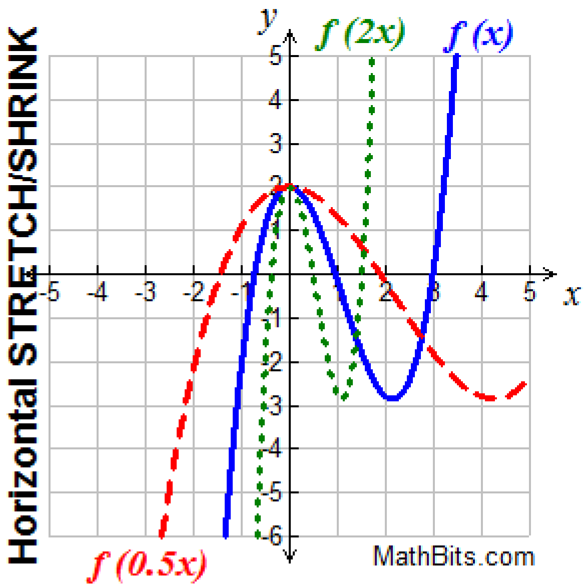
$f(kx)$ stretches/shrinks $f(x)$ horizontally

A **horizontal stretching** is the stretching of the graph away from the $y$-axis

A **horizontal compression** (or **shrinking**) is the squeezing of the graph toward the $y$-axis.

- if $k > 1$, the graph of $y = k \cdot f(x)$ is the graph of $f(x)$ **horizontally shrunk (or compressed)** by dividing each of its $x$-coordinates by $k$.
- if $0 < k < 1$ (*a fraction*), the graph is $f(x)$ **horizontally stretched** by dividing each of its $x$-coordinates by $k$.
- if $k$ should be negative, the horizontal stretch or shrink is followed by a reflection in the $y$-axis.

Notice that the "roots" on the graph have now moved, but the $y$-intercept stays in its same initial position for all graphs. The graph gets "taffy pulled" left and right from the locking $y$-intercept. The $x$-values change.




"Divide x-coordinates"
(x, y) becomes (x/k, y)
"horizontal dilation"

| Transformations of Function Graphs | |
|---|---|
| $-f(x)$ | reflect $f(x)$ over the $x$-axis |
| $f(-x)$ | reflect $f(x)$ over the y-axis |
| $f(x) + k$ | shift $f(x)$ up $k$ units |
| $f(x) - k$ | shift $f(x)$ down $k$ units |
| $f(x + k)$ | shift $f(x)$ left $k$ units |
| $f(x - k)$ | shift $f(x)$ right $k$ units |
| $k \cdot f(x)$ | multiply $y$-values by $k$ |
| $f(kx)$ | divide $x$-values by $k$ |


| Transformations of Function Graphs | |
|---|---|
| $-f(x)$ | reflect $f(x)$ over the $x$-axis |
| $f(-x)$ | reflect $f(x)$ over the y-axis |
| $f(x) + k$ | shift $f(x)$ up $k$ units |
| $f(x) - k$ | shift $f(x)$ down $k$ units |
| $f(x + k)$ | shift $f(x)$ left $k$ units |
| $f(x - k)$ | shift $f(x)$ right $k$ units |
| $k \cdot f(x)$ | multiply $y$-values by $k$ |
| $f(kx)$ | divide $x$-values by $k$ |