Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Even and Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Sum and Difference Identities

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle and Half-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

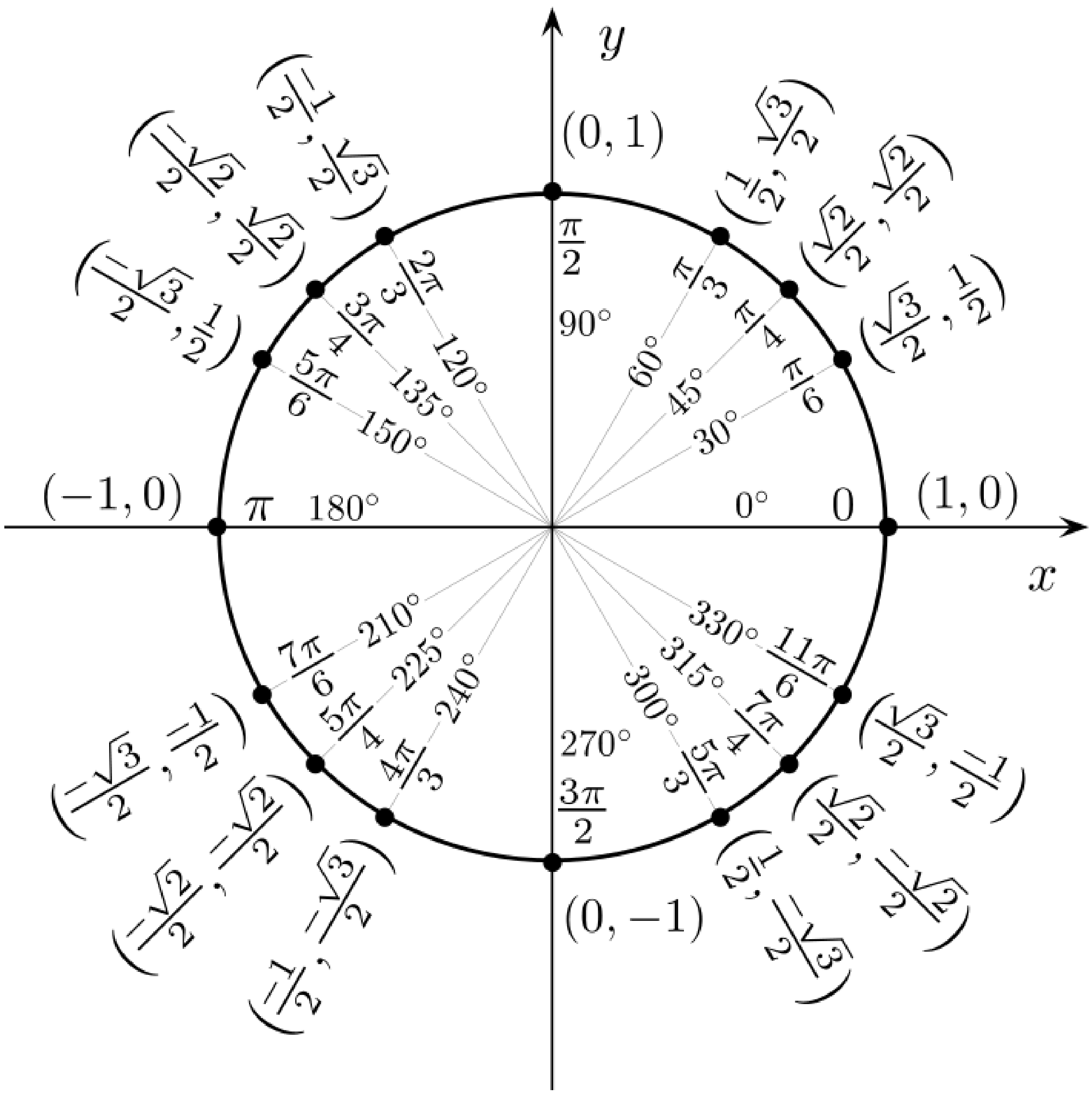
$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$



y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
π/2
90°
π
180°
0
0°
3π/2
270°
π/6
30°
π/4
45°
π/3
60°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
5π/3
300°
7π/4
315°
11π/6
330°
(√3/2, 1/2)
(√2/2, √2/2)
(1/2, √3/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

Sum and Difference Identities

NOTES

Write your questions here!

Is it true?

$\sin(45° + 30°) = \sin 45° + \sin 30°$

Ex 1:

Sum/Difference Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Ex 2:

Ex 3:

Ex 4: Write the expression as the sine, cosine, or tangent of an angle.

Ex 5: Find sin (x - y) given the following:

Ex 6: Is the equation an identity?

Now, summarize your notes here!

SUMMARY:

Sum and Difference Identities

PRACTICE

Directions: Tell whether each statement is true or false.

1) $\sin 75 = \sin 50 \cos 25 - \cos 25 \sin 25$	2) $\cos 15 = \cos 60 \cos 45 + \sin 60 \sin 45$	3) $\tan 225 = \dfrac{\tan 180 - \tan 45}{1 + \tan 180 \tan 45}$

Directions: Write the expression as the sine, cosine or tangent of an angle.

4) $\sin 42 \cos 17 - \cos 42 \sin 17$	5) $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47}$	6) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

Directions: Use the sum or difference identity to find the exact value.

7) $\tan 195°$	8) $\cos 255°$
9) $\sin 165°$	10) $\cos\frac{13\pi}{12}$

11) $\sin\frac{5\pi}{12}$

12) $\tan\frac{\pi}{12}$

Directions: Find the exact value.

13) $\sin(\propto - \beta)$

Given: $\cos\propto = \frac{3}{5}, where\ 0 < \propto < \frac{\pi}{2}$

$\tan\beta = \frac{12}{5}, where\ 0 < \beta < \frac{\pi}{2}$

14) $\tan(x - y)$

Given: $\cos \text{x} = \frac{7}{25}, where\ 0° < x < 90°$

$\cos y = -\frac{4}{5}, where\ 90° < y < 180°$

15) $\sin(\propto + \beta)$

Given: $\sin\propto = \frac{4}{5}, where\ \propto is\ in\ Quadrant\ I$

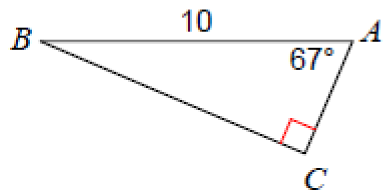
$\cos\beta = -\frac{24}{25}, where\ \beta\ is\ in\ Quadrant\ III$

16) $\cos(x + y)$

Given: $\cos x = \frac{15}{17}, where\ \frac{3\pi}{2} < x < 2\pi$

$\tan y = \frac{4}{3}, where\ \pi < y < \frac{3\pi}{2}$

Directions: Is the equation an identity? Explain using the sum or difference identities	
17) $\cos(x-\pi)=\cos x$	18) $\sin(x-\pi)=\sin x$

REVIEW SKILLZ: Directions: Solve each triangle.	
1) 	2) 

11.3 Application and Extension

1) Find the exact value.

$\cos 285°$

2) Find the exact value.

$\cos(x+y)$

Given: $\cos x=\frac{15}{17}, where\ \frac{3\pi}{2}<x<2\pi$

$\tan y=\frac{4}{3}, where\ \pi<y<\frac{3\pi}{2}$

3) Verify the following DOUBLE ANGLE IDENTITIES. (Hint….sin (2x) = sin (x + x)

a) $\sin(2x) = 2\sin x \cos x$

b) $\cos(2x) = 2\,cos^2 x - 1$

5) When a wave travels through a taut string (like guitar string), the displacement y of each point on the string depends on the time t and the point's position x. The equation of a standing wave can be obtained by adding the displacements of two waves traveling in opposite directions. Suppose two waves can be modeled by the following equations:

$$y_1 = A\cos\left(\frac{2\pi t}{3} - \frac{2\pi x}{5}\right) \qquad y_2 = A\cos\left(\frac{2\pi t}{3} + \frac{2\pi x}{5}\right)$$

Find $y_1 + y_2$

6) Mr. Sullivan has been carrying the other Algebros on his back for the last several years. He knows from Mr. Rahn's physics' class that the force F (in pounds) on a person's back when he bends over at an angle θ is:

$$F = \frac{0.6W\sin(\theta + 90°)}{\sin 12°}$$

Simplify the above formula.

Sum and Difference Identities

PRACTICE

Directions: Tell whether each statement is true or false.

1) $\sin 75 = \sin 50 \cos 25 - \cos 25 \sin 25$

FALSE

2) $\cos 15 = \cos 60 \cos 45 + \sin 60 \sin 45$

TRUE

3) $\tan 225 = \dfrac{\tan 180 - \tan 45}{1 + \tan 180 \tan 45}$

FALSE

Directions: Write the expression as the sine, cosine or tangent of an angle.

4) $\sin 42 \cos 17 - \cos 42 \sin 17$

$\sin 25^\circ$

5) $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47}$

$\tan 66^\circ$

6) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\frac{4\pi}{12}$, $\frac{3\pi}{12}$, $\frac{4\pi}{12}$, $\frac{3\pi}{12}$

$\frac{4\pi}{12} - \frac{3\pi}{12}$

$\boxed{\cos\frac{\pi}{12}}$

Directions: Use the sum or difference identity to find the exact value.

7) $\tan 195°$ $\quad 150 + 45$

$$\frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45} = \frac{3\left(-\frac{\sqrt{3}}{3} + 1\right)}{3\left(1 - -\frac{\sqrt{3}}{3}(1)\right)}$$

$$= \frac{-\sqrt{3} + 3}{3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})} = \frac{-3\sqrt{3} + 3 + 9 - 3\sqrt{3}}{9 - 3}$$

$$= \frac{12 - 6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$$

8) $\cos 255°$ $\quad 300 - 45$

$$= \cos 300 \cos 45 + \sin 300 \sin 45$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{-\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{-\sqrt{6}}{4}$$

$$\boxed{= \frac{\sqrt{2} - \sqrt{6}}{4}}$$

9) $\sin 165°$ $\quad 135 + 30$

$$= \sin 135 (\cos 30) + \cos 135 (\sin 30)$$

$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$$

$$\boxed{= \frac{\sqrt{6} - \sqrt{2}}{4}}$$

10) $\cos \frac{13\pi}{12}$ $\quad \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$

$$\cos\frac{3\pi}{4} \cdot \cos\frac{\pi}{3} - \sin\frac{3\pi}{4} \cdot \sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\boxed{= \frac{-\sqrt{2} - \sqrt{6}}{4}}$$

11) $\sin\frac{5\pi}{12}$ $\quad \frac{8\pi}{12}-\frac{3\pi}{12} = \frac{2\pi}{3}-\frac{\pi}{4}$

$\sin\left(\frac{2\pi}{3}\right)\cos\frac{\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \left(\frac{-\sqrt{2}}{4}\right)$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

12) $\tan\frac{\pi}{12}$ $\quad \frac{4\pi}{12}-\frac{3\pi}{12} = \frac{\pi}{3}-\frac{\pi}{4}$

$= \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}(1)} =$

$= \frac{\sqrt{3}-1}{1+\sqrt{3}}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$

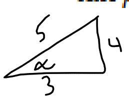
$= \frac{2\sqrt{3}-4}{-2} = \frac{2\sqrt{3}}{-2} + \frac{-4}{-2} = \boxed{-\sqrt{3}+2}$

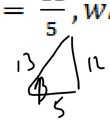
Directions: Find the exact value.

13) $\sin(\alpha-\beta)$

Given: $\cos\alpha = \frac{3}{5}$, *where* $0 < \alpha < \frac{\pi}{2}$

$\tan\beta = \frac{12}{5}$, *where* $0 < \beta < \frac{\pi}{2}$





$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

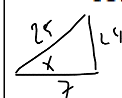
$= \frac{4}{5}\left(\frac{5}{13}\right) - \frac{3}{5}\left(\frac{12}{13}\right)$

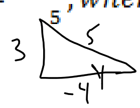
$= \frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$

14) $\tan(x-y)$

Given: $\cos x = \frac{7}{25}$, *where* $0° < x < 90°$

$\cos y = -\frac{4}{5}$, *where* $90° < y < 180°$





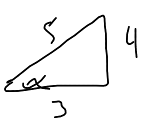
$= \frac{\left(\frac{24}{7}\right) - \left(\frac{-3}{4}\right)}{1+\left(\frac{24}{7}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{117}{28}}{1+\left(\frac{-72}{28}\right)} = \frac{\frac{117}{28}}{\frac{-44}{28}}$

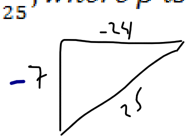
$= \frac{117}{28}\left(\frac{28}{-44}\right) = \boxed{-\frac{117}{44}}$

15) $\sin(\alpha+\beta)$

Given: $\sin\alpha = \frac{4}{5}$, *where* α *is in Quadrant I*

$\cos\beta = -\frac{24}{25}$, *where* β *is in Quadrant III*





$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

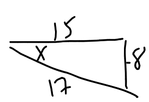
$= \frac{4}{5}\left(\frac{-24}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{-7}{25}\right)$

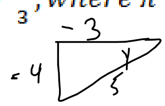
$= \frac{-96}{125} + \frac{-21}{125} = \boxed{\frac{-117}{125}}$

16) $\cos(x+y)$

Given: $\cos x = \frac{15}{17}$, *where* $\frac{3\pi}{2} < x < 2\pi$

$\tan y = \frac{4}{3}$, *where* $\pi < y < \frac{3\pi}{2}$





$\cos x\cos y - \sin x\sin y$

$= \left(\frac{15}{17}\right)\left(\frac{-3}{5}\right) - \frac{-8}{17}\left(\frac{-4}{5}\right)$

$= \frac{-45}{85} - \frac{32}{85} = \boxed{\frac{-77}{85}}$

Directions: Is the equation an identity? Explain using the sum or difference identities

17) $\cos(x - \pi) = \cos x$

$\cos x \cos \pi + \sin x \sin \pi = \cos x$

$\cos x(-1) + \sin x(0) = \cos x$

$-\cos x = \cos x$

NO ITS NOT AN IDENTITY

18) $\sin(x - \pi) = \sin x$

$\sin x \cos \pi - \cos x \sin \pi = \sin x$

$= \sin x \cdot -1 - \cos x \cdot 0 = \sin x$

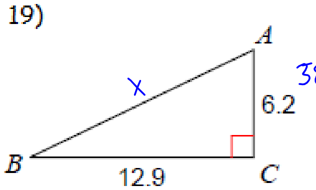
$= -\sin x - 0 = \sin x$

$= -\sin x \neq \sin x$

NOT AN IDENTITY

Directions: Solve each triangle.

19)



$6.2^2 + 12.9^2 = x^2$

$38.44 + 166.41 = x^2$

$\sqrt{204.85} = \sqrt{x^2}$

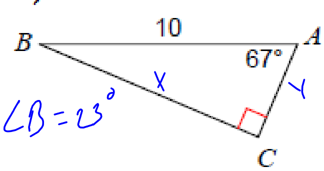
$14.3 = x$

$\tan B = \frac{6.2}{12.9}$

$\angle B = 25.7^0$

$\angle A = 64.3^0$

20)



$\angle B = 23^0$

$\sin 67 = \frac{x}{10}$

$9.2 = x$

$\cos 67 = \frac{y}{10}$

$3.9 = y$

You must complete this before retaking the MC again. Remember it is all about LEARNING so take your time and learn how to do these skills. If you need help please ask!

NAME:_____________________

Directions: Write the expression as the sine, cosine or tangent of an angle.		
1) $\sin 27 \cos 24 - \cos 27 \sin 24$	2) $\frac{\tan 24+\tan 13}{1-\tan 24 \tan 13}$	3) $\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

Directions: Use the sum or difference identity to find the exact value.	
4) $\cos 255°$	5) $\sin\ 105°$
6) $\sin\frac{13\pi}{12}$	7) $\tan\frac{5\pi}{12}$

Directions: Find the exact value.	
8) $\tan(\propto - \beta)$ Given: $\cos \propto = \frac{5}{13}, where\ 0 < \propto < \frac{\pi}{2}$ $\tan\beta = \frac{3}{4}, where\ 0 < \beta < \frac{\pi}{2}$	9) $\sin(x - y)$ Given: $\cos x = \frac{7}{25}, where\ 0° < x < 90°$ $\cos y = -\frac{3}{5}, where\ 90° < y < 180°$
10) $\cos(\propto + \beta)$ Given: $\sin \propto = \frac{4}{5}, where\ \propto is\ in\ Quadrant\ I$ $\tan\beta = \frac{5}{12}, where\ \beta\ is\ in\ Quadrant\ III$	11) $\sin(x + y)$ Given: $\cos x = \frac{15}{17}, where\ \frac{3\pi}{2} < x < 2\pi$ $\tan y = \frac{4}{3}, where\ \pi < y < \frac{3\pi}{2}$

ANSWERS TO CORRECTIVE ASSIGNMENT:

Make sure you check all your answers and make sure you KNOW how to do all of them. You could simply copy answers but that's not the point. The point is that you have to learn how to do this so please make sure that for any you don't understand you get help BEFORE taking the Mastery Check again.

1) $\sin 3°$ 2) $\tan 37°$ 3) $\cos\frac{\pi}{12}$ 4) $\frac{-\sqrt{6}+\sqrt{2}}{4}$ 5) $\frac{\sqrt{6}+\sqrt{2}}{4}$ 6) $\frac{\sqrt{2}-\sqrt{6}}{4}$ 7) $\sqrt{3}+2$ 8) $\frac{33}{56}$ 9) $-\frac{4}{5}$ 10) $-\frac{16}{65}$ 11) $-\frac{36}{85}$

Name:____**Date:**______________________

Worksheet: Sum and Difference Identities

- $\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
- $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
- $\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

Example 1: Find the *exact* value of cos 15°. 1. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Example 2: Find the *exact* value of $\sin \dfrac{13\pi}{12}$. 2. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

Example 3: Find the *exact* value of $\tan \dfrac{17\pi}{12}$. 3. $2+\sqrt{3}$

Example 4: There is no formula given for the cotangent of a sum or difference. Describe how you would find the exact value of cot 15°.

Example 5: Write each as a single trigonometric function:

a. $\cos 4x \cos 3x + \sin 4x \sin 3x$ a.______________

b. $\sin 2x \cos x - \sin x \cos 2x$ b.______________

c. $\dfrac{\tan 5x - \tan 4x}{1 + \tan 5x \tan 4x}$ c.______________

d. $\sin 10° \cos 5° + \cos 10° \sin 5°$ d.______________

e. $\cos 37° \cos 22° - \sin 37° \sin 22°$ e.______________

For questions 1-6, find the *exact* value for each of the following:

1. $\sin 15°$ 1.________

2. $\cos\frac{5\pi}{12}$ 2.________

3. $\tan 105°$ 3.________

4. $\sin\frac{19\pi}{12}$ 4.________

5. $\cos 255°$ 5.________

6. $\tan\frac{13\pi}{12}$ 6.________

For questions 7-9, write each expression as a single trigonometric function:

7. $\sin 3x\cos 2x + \cos 3x\sin 2x$ 7.________

8. $\cos 5x\cos x - \sin 5x\sin x$ 8.________

9. $\cos 15°\cos 75° - \sin 15°\sin 75°$ 9.________

For questions 10 and 11, find the solutions in the interval $[0, 2\pi)$.

10. $\sin 2x\cos x + \cos 2x\sin x = -1/2$ 10.________

11. $\cos 2x\cos x - \sin 2x\sin x = \frac{-\sqrt{3}}{2}$ 11.________

For question 12, find **all** degree solutions.

12. $\sin 3x\cos 2x + \cos 3x\sin 2x = 1$ 12.________

Text (pp. 468-471): #s 11-22 all, 59-61 all

Name:____

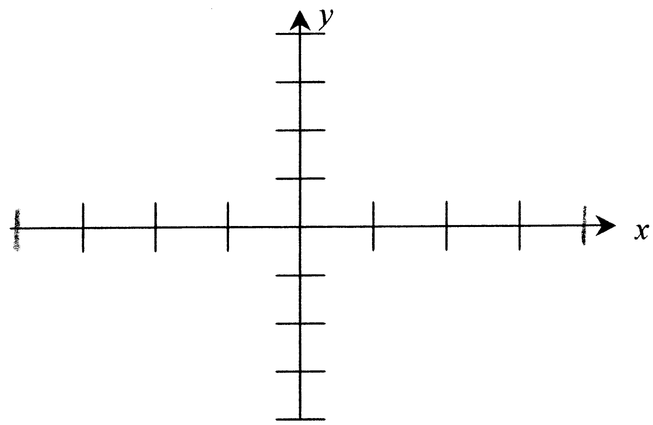
Activity: Verifying Sum and Difference Identities

On the axis provided, NEATLY sketch the graph given as y_1. Use RADIAN MODE and ZOOM 7.
Identify the graph as y_2.
Then prove *algebraically*, that $y_1 = y_2$.

1. $y_1 = \cos(\pi + x)$

$y_2 =$ ____________

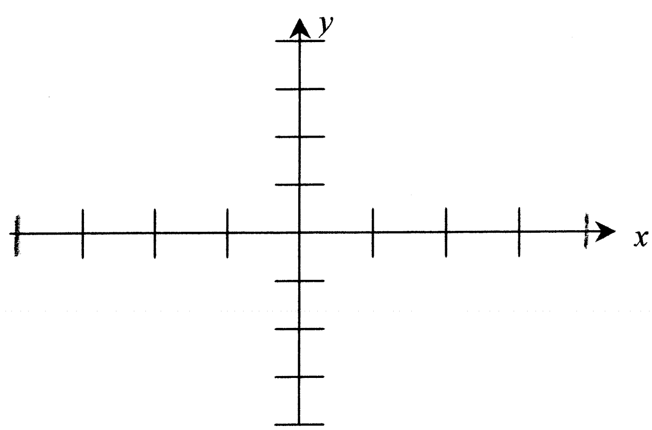
Proof that that $y_1 = y_2$



2. $y_1 = \sin(\pi + x)$

$y_2 =$ ____________

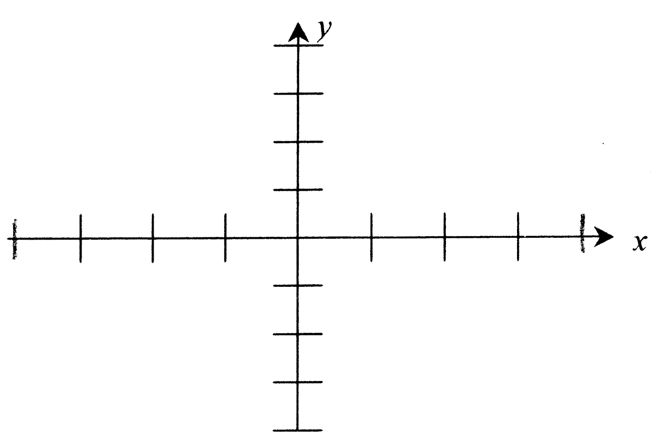
Proof that that $y_1 = y_2$



3. $y_1 = \tan(x - \pi)$ **show asymptotes*

$y_2 =$ ____________

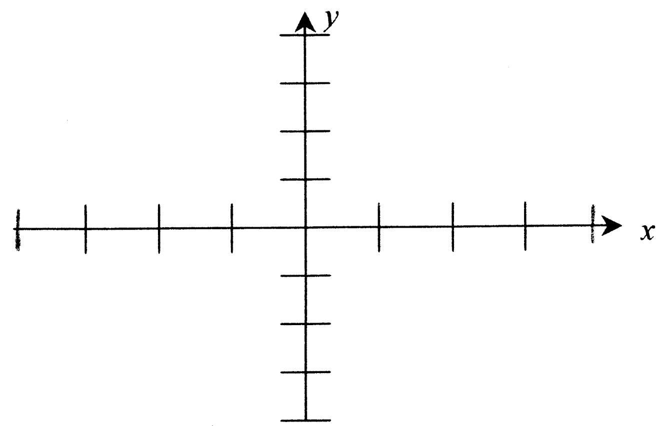
Proof that that $y_1 = y_2$



4. $y_1 = \sin(x + \pi/2)$

$y_2 =$ ______________

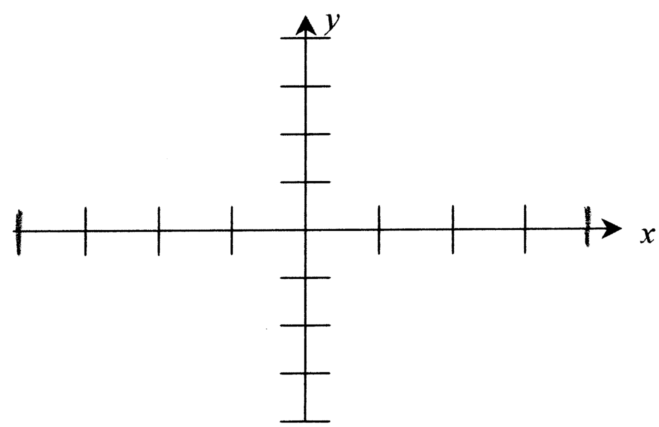
Proof that that $y_1 = y_2$



5. $y_1 = \cos(x - \pi/2)$

$y_2 =$ ______________

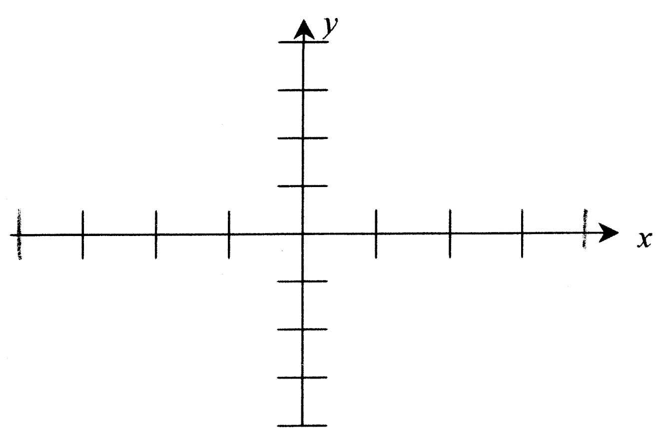
Proof that that $y_1 = y_2$



6. $y_1 = \tan(\pi + x)$ *show asymptotes*

$y_2 =$ ______________

Proof that that $y_1 = y_2$



Name:____ **Date:**____________________

Precalculus Worksheet – Angle Between Two Lines

If θ is the angle formed by the intersecting of 2 lines, l_1 and l_2, where m_1 is the slope of l_1 and m_2 is the slope of l_2, we have the formula:

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1} \text{ where } 0° \le \theta \le 180°$$

<u>Notes</u>: 1. l_2 is the line making the <u>greater</u> angle with the positive x-axis.

2. $m_1 m_2 \neq -1$ in this formula. But if $m_1 m_2 = -1$, we already know θ.
3. Neither line may be vertical in this formula. If it is, we find θ by other means.
4. If $\tan\theta \ge 0$, $\theta = \arctan\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$, but if $\tan\theta < 0$, we must add 180°, since $0° \le \theta \le 180°$.

<u>Problems:</u>

1. Find the tangent of the angle between the lines with slopes:

   a. $\frac{1}{2}, \frac{2}{3}$ *b.* $\frac{-3}{4}, \frac{-5}{2}$ *c.* $\frac{-2}{7}, \frac{5}{3}$ *d.* m and 0 (for any m)

2. To the nearest degree, find the angle between the lines whose slopes are:

   a. $\frac{5}{2}, \frac{2}{3}$ *b.* -1.3, 0.6 *c.* $\frac{1}{4}, 2$ *d.* no slope, $\frac{-1}{2}$

3. The tangent of the angle between 2 lines is $\frac{-4}{9}$ and the slope of the line with the smaller angle is $\frac{3}{7}$. Find the slope of the other line.

4. To the nearest degree, find the interior angles of the triangle with vertices (-2, -3), (-5, 4) and (6, 1).

5. Show that the triangle with vertices (-2, 3), (6, 9) and (4, 11) is isosceles by using the formula for θ.

6. If q is the line containing the points (2, 1) and (4, -3), and l forms an angle of 45° with q, find the slope of l.

<u>Answers</u>

1. *a.* $\frac{1}{8}$ *b.* $\frac{14}{23}$ *c.* $\frac{-41}{11}$ *d.* m

2. *a.* 34.51°, b. 96.61°, c. 49.4°, d. 63.43°

3. $m_2 = \frac{-1}{75}$

4. 87°, 42°, 52°

5. Two of the angles are 81.87°

6. Two possible answers: $m = 3$ or $m = \frac{-1}{3}$

Name:______________________________ Date:______________

Worksheet: Double Angle Identities

• $\sin(2x) = 2\sin x\cos x$	• $\cos(2x) = \cos^2 x - \sin^2 x$ $= 1 - 2\sin^2 x$ $= 2\cos^2 x - 1$	• $\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$

I. Write as a single, simplified trig expression.

1. $2\sin A\cos A$
2. $\dfrac{2\tan B}{1-\tan^2 B}$
3. $1-2\sin^2 C$
4. $2\sin 105°\cos 105°$
5. $\cos^2 15° - \sin^2 15°$
6. $\dfrac{2\tan 67.5°}{1-\tan^2 67.5°}$
7. $2\cos^2 112.5° - 1$
8. $\sin 75°\cos 15° + \sin 15°\cos 75°$
9. $\dfrac{\tan 52.5° + \tan 7.5°}{1-\tan 52.5°\tan 7.5°}$
10. $\cos 127.5°\cos 7.5° + \sin 127.5°\sin 7.5°$
11. $\sin\left(\dfrac{\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right)$

II. Find the *exact* values of sin2*x*, cos2*x* and tan2*x* without using a calculator.

12. $\sin x = \dfrac{5}{13}, x$ in QII $\left\{\dfrac{-120}{169}, \dfrac{119}{169}, \dfrac{-120}{119}\right\}$

13. $\cos x = \dfrac{-2}{3}, x$ in QII $\left\{\dfrac{-4\sqrt{5}}{9}, \dfrac{-1}{9}, 4\sqrt{5}\right\}$

14. $\tan x = \dfrac{1}{2}, x$ in QIII $\left\{\dfrac{4}{5}, \dfrac{3}{5}, \dfrac{4}{3}\right\}$

15. $\cot x = -4, x$ in QIV $\left\{\dfrac{-8}{17}, \dfrac{15}{17}, \dfrac{-8}{15}\right\}$

III. Solving (#3 requires use of the quadratic formula)

16. $\sin 2x = \sin x$; $[0°, 360°)$

$\{0°, 180°, 60°, 300°\}$

17. $\sin 2x - \cos x = 0$; $[0°, 360°)$

$\{90°, 270°, 30°, 150°\}$

18. $\cos 2x + 2\sin x = 0$; $[0°, 360°)$

$\{201.47°, 338.53°\}$

HW: {5.4} pp. 475-476 #s 5-10all; 23-25all

Name:______________________________ Date:______________

Precalculus: More Identities - From Double Angle to Power Reducing to Half-Angle

Power Reducing Identities: $\begin{cases} \sin^2 u = \dfrac{1-\cos 2u}{2} \\ \cos^2 u = \dfrac{1+\cos 2u}{2} \\ \tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u} \end{cases}$

1. Solve for $\sin^2 u$: $\cos(2u) = 1 - 2\sin^2 u$

2. Solve for $\cos^2 u$: $\cos(2u) = 2\cos^2 u - 1$

3. Determine $\tan^2 u$ using the quotient identity.

Half-Angle Identities: $\begin{cases} \sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}, \quad \cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}} \\ \tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{1-\cos A}{\sin A} = \dfrac{\sin A}{1+\cos A} \end{cases}$

4. Find $\sin u$: $\sin^2 u = \dfrac{1-\cos 2u}{2}$

5. Find $\cos u$: $\cos^2 u = \dfrac{1+\cos 2u}{2}$

6. Find $\tan u$: $\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

Let $2u = A$, $u = \dfrac{A}{2}$

Problems: {5.4} pp. 475-476 #s 31-36all, 50, 52

Name:______________________________ Date:______________

Trigonometry: Review for Identities Quiz

ANSWERS

1. Use a **sum/difference formula** to find the <u>exact</u> value of $\cos\frac{11\pi}{12}$. Show all work.

$\frac{-\sqrt{2}-\sqrt{6}}{4}$

2. Use a **half-angle identity** to determine the <u>exact</u> value of $\tan 67.5°$. Show all work.

$1+\sqrt{2}$

3. Solve over the interval **[0, 2π)**: $\sin 2x + 2\cos x = 0$

$\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

4. Solve over the interval **[0°, 360°)**: $\cos 2x + 2 = -4\cos x - 2\cos^2 x$

$\{120°, 240°\}$

5. Find the *acute* angle formed by the lines $4x+3y+1=0$ and $2x-4y+1=0$

$\theta = 79.70°$

6. Find the *acute* angle formed by the lines $x=10$ and $y=\frac{-2}{9}x+3$.

$\theta = 77.47°$

7. Use the **double-angle identities** to find the <u>exact</u> values of $\sin 2x$, $\cos 2x$, and $\cot 2x$ if $\sec x = -\sqrt{5}$ and x lies in quadrant III. Draw a diagram.

$\left\{\frac{4}{5}, \frac{-3}{5}, \frac{-3}{4}\right\}$

8. Matching (Some of the expressions on the right may be used more than once or not at all.)

a. $\sin(\alpha - \beta)$

b. $\cos(\alpha + \beta)$

c. $\sin(180° + \beta)$

d. $\sin(180° - \beta)$

e. $\cos(180° + \beta)$

f. $\sin(\alpha + \beta)$

g. $\cos(90° - \beta)$

h. $\cos(\alpha - \beta)$

i. $\sin\beta$

ii. $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

iii. $-\cos\beta$

iv. $\cos\alpha\cos\beta + \sin\alpha\sin\beta$

v. $\sin\alpha\cos\beta - \cos\alpha\sin\beta$

vi. $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

vii. $-\sin\beta$

viii. $\cos\beta$

9. Which expressions are equal to $\sin 15°$ **?** (There may be more than one correct choice.)

A. $\sin 45°\cos 30° + \cos 45°\sin 30°$

B. $\sin 45°\cos 30° - \cos 45°\sin 30°$

C. $\sin 60°\cos 45° - \cos 60°\sin 45°$

D. $\cos 60°\cos 45° - \sin 60°\sin 45°$

10. Matching

a. $\sin\dfrac{\beta}{2}$

b. $\cos 2\beta$

c. $\cos\dfrac{\beta}{2}$

d. $\sin 2\beta$

i. $2\sin\beta\cos\beta$

ii. $1 - 2\sin^2\beta$

iii. $\cos^2\beta - \sin^2\beta$

iv. $\pm\sqrt{\dfrac{1+\cos\beta}{2}}$

v. $\pm\sqrt{\dfrac{1-\cos\beta}{2}}$

11. Thinking Graphically – which equations have no solution?

A. $\sin x = 1$

B. $\tan x = 0.001$

C. $\sec x = \dfrac{1}{2}$

D. $\csc x = -3$

E. $\cos x = 1.01$

F. $\cot x = -1000$

G. $\cos x + 2 = -1$

H. $\sec x - 1.5 = 0$

I. $\sin x - 0.009 = 0.99$

Practice with Sum and Difference Identities

Write each expression as the sine, cosine, or tangent of a single angle.

_____________________ 1. $\cos 80° \cos 20° + \sin 80° \sin 20°$

_____________________ 2. $\sin 30° \cos 45° + \cos 30° \sin 45°$

_____________________ 3. $\cos 20° \cos 45° - \sin 20° \sin 45°$

_____________________ 4. $\frac{\tan 90° - \tan 10°}{1 + \tan 90° \tan 10°}$

_____________________ 5. $\sin 25° \cos 20° - \cos 25° \sin 20°$

_____________________ 6. $\frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$

_____________________ 7. $\sin\frac{\pi}{4} \cos\frac{\pi}{3} - \cos\frac{\pi}{4} \sin\frac{\pi}{3}$

_____________________ 8. $\cos\frac{\pi}{2} \cos\frac{2\pi}{3} + \sin\frac{\pi}{2} \sin\frac{2\pi}{3}$

_____________________ 9. $\frac{\tan\frac{\pi}{9} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{9} \tan\frac{\pi}{4}}$

_____________________ 10. $\sin 105° \cos 85° - \cos 105° \sin 85°$

_____________________ 11. $\cos 45° \cos 60° + \sin 45° \sin 60°$

_____________________ 12. $\cos 45° \cos 60° - \sin 45° \sin 60°$

Write each expression as a sine, cosine, or tangent of a sum or difference of special value angles.

_____________________ 13. $\sin 105°$

_____________________ 14. $\cos\frac{\pi}{12}$

_____________________ 15. $\tan\frac{7\pi}{12}$

_____________________ 16. $\cos(-75°)$

_____________________ 17. $\sin 165°$

______________________ 18. $\cos 195\,°$

______________________ 19. $\tan 285°$

______________________ 20. $\sin \frac{13\pi}{12}$

Determine if each equation is true or false. If the statement is false, highlight the incorrect section.

______________________ 21. $\cos 57° = \cos(40° + 17°)$

______________________ 22. $\sin 75° = \sin 50° \cos 25° - \cos 50° \sin 25°$

______________________ 23. $\tan 45° = \frac{\tan 40° + \tan 5°}{1 + \tan 40° \tan 5°}$

______________________ 24. $\sin 40° = \sin 50° - \sin 10°$

______________________ 25. $\cos 65° = \cos 35° \cos 30° + \sin 35° \sin 30°$

______________________ 26. $\sin 105° = \sin 90° \cos 15° + \sin 15° \cos 90°$

______________________ 27. $\tan 75° = \frac{\tan 90° - \tan 15°}{1 + \tan 15° \tan 90°}$

______________________ 28. $\tan 75° = \frac{\tan 80° - \tan 5°}{1 + \tan 70° \tan 5°}$

______________________ 29. $\cos 60° = \cos 20° + \cos 40°$

______________________ 30. $\sin 25° = \sin 10° \cos 15° + \sin 10° \cos 15°$

Practice with Sum and Difference Identities KEY

Write each expression as the sine, cosine, or tangent of a single angle.

$\cos 60°$ ________ 1. $\cos 80° \cos 20° + \sin 80° \sin 20°$

$\sin 75°$ ________ 2. $\sin 30° \cos 45° + \cos 30° \sin 45°$

$\cos 65°$ ________ 3. $\cos 20° \cos 45° - \sin 20° \sin 45°$

$\tan 80°$ ________ 4. $\frac{\tan 90° - \tan 10°}{1 + \tan 90° \tan 10°}$

$\sin 5°$ ________ 5. $\sin 25° \cos 20° - \cos 25° \sin 20°$

$\tan 165°$ ________ 6. $\frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$

$\sin\left(-\frac{\pi}{12}\right)$ ________ 7. $\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$\cos\left(-\frac{\pi}{6}\right)$ ________ 8. $\cos\frac{\pi}{2}\cos\frac{2\pi}{3} + \sin\frac{\pi}{2}\sin\frac{2\pi}{3}$

$\tan\frac{5\pi}{36}$ ________ 9. $\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{9}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{9}}$

$\sin 20°$ ________ 10. $\sin 105° \cos 85° - \cos 105° \sin 85°$

$\cos(-15°)$ ________ 11. $\cos 45° \cos 60° + \sin 45° \sin 60°$

$\cos 105°$ ________ 12. $\cos 45° \cos 60° - \sin 45° \sin 60°$

Write each expression as a sine, cosine, or tangent of a sum or difference of special value angles.

$\sin(60° + 45°)$ ________ 13. $\sin 105°$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ ________ 14. $\cos\frac{\pi}{12}$

$\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ ________ 15. $\tan\frac{7\pi}{12}$

$\cos(45° - 120°)$ ________ 16. $\cos(-75°)$

$\sin(120° + 45°)$ ________ 17. $\sin 165°$

$\cos(150° + 45°)$ 18. $\cos 195°$

$\tan(240° + 45°)$ 19. $\tan 285°$

$\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ 20. $\sin \frac{13\pi}{12}$

Determine if each equation is true or false. If the statement is false, highlight the incorrect section.

T 21. $\cos 57° = \cos(40° + 17°)$

F 22. $\sin 75° = \sin 50° \cos 25° - \cos 50° \sin 25°$

F 23. $\tan 45° = \frac{\tan 40° + \tan 5°}{1 + \tan 40° \tan 5°}$

F 24. $\sin 40° = \sin 50° - \sin 10°$ Can't just distribute.

F 25. $\cos 65° = \cos 35° \cos 30° + \sin 35° \sin 30°$

T 26. $\sin 105° = \sin 90° \cos 15° + \sin 15° \cos 90°$

T 27. $\tan 75° = \frac{\tan 90° - \tan 15°}{1 + \tan 15° \tan 90°}$

F 28. $\tan 75° = \frac{\tan 80° - \tan 5°}{1 + \tan 70° \tan 5°}$

F 29. $\cos 60° = \cos 20° + \cos 40°$ Can't just distribute.

F 30. $\sin 25° = \sin 10° \cos 15° + \sin 10° \cos 15°$

REVIEW—SUM/DIFFERENCE IDENTITIES **NAME**______________________________

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^o \cos 43^o + \cos 97^o \sin 43^o$

2. $\cos 72^o \cos 130^o + \sin 72^o \sin 130^o$

3. $\dfrac{\tan 140^o - \tan 60^\circ}{1 + \tan 140^o \tan 60^\circ}$

4. $\sin\dfrac{\pi}{5}\cos\dfrac{2\pi}{3} - \cos\dfrac{\pi}{5}\sin\dfrac{2\pi}{3}$

5. $\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{7} - \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{7}$

6. $\dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. $\tan(-105^o)$

8. $\sin 345^o$

#9-11. Given: $\csc\alpha = \dfrac{13}{5}$, $\dfrac{\pi}{2} \le \alpha \le \pi$, and $\tan\beta = -\dfrac{3}{4}$, $\dfrac{3\pi}{2} \le \beta \le 2\pi$, find the following:

9. $\sin(\alpha - \beta)$

10. $\cos(\beta + \alpha)$

11. $\tan(\alpha - \beta)$

#12-13. If $\sin\theta = -\dfrac{3}{5}$ and θ is in the third quadrant, find the following:

12. $\cos(\theta + \dfrac{\pi}{3})$

13. $\tan 2\theta$

#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\dfrac{3\pi}{2} + x) = -\cos x$

16. $\cos(30^0 - x) + \cos(30^o + x) = \sqrt{3}\cos x$

17. $\dfrac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta} = \cot\alpha - \cot\beta$

18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

19. $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

20. $\tan(x+\pi)+2\sin(x+\pi)=0$

21. $\sin\left(x+\frac{\pi}{2}\right)-\cos\left(x+\frac{3\pi}{2}\right)=0$

Answers: 1. $\sin 190^o$ 2. $\cos 10^o$ 3. $\tan 80^o$ 4. $-\sin\left(\frac{7\pi}{15}\right)$

5. $\cos\left(\frac{13\pi}{42}\right)$ 6. $\tan\frac{7\pi}{12}$ 7. $2+\sqrt{3}$ 8. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 9. $-\frac{16}{65}$

10. $-\frac{33}{65}$ 11. $\frac{16}{63}$ 12. $\frac{-4+3\sqrt{3}}{10}$ 13. $\frac{24}{7}$ 19. $\frac{\pi}{3}, \frac{5\pi}{3}$

20. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 21. $\frac{\pi}{4}, \frac{5\pi}{4}$

School:Aisha bint abi bakre Secondary

Name________________________________

Angle Sum/Difference Identities

Date________________ Period____

Use the angle sum identity to find the exact value of each.

1) $\cos 105°$

2) $\sin 195°$

3) $\cos 195°$

4) $\cos 165°$

5) $\cos 285°$

6) $\cos 255°$

7) $\sin 105°$

8) $\sin 285°$

9) $\cos 75°$

10) $\sin 255°$

Use the angle difference identity to find the exact value of each.

11) $\cos 75°$

12) $\cos -15°$

13) $\tan 75°$

14) $\cos 15°$

15) $\tan -105°$

16) $\sin 105°$

17) $\tan 15°$

18) $\sin 15°$

19) $\tan -15°$

20) $\sin -75°$

Use the angle sum or difference identity to find the exact value of each.

21) $\sin -105°$

22) $\cos 195°$

23) $\cos \frac{7\pi}{12}$

24) $\tan \frac{13\pi}{12}$

25) $\sin \frac{\pi}{12}$

26) $\cos -\frac{7\pi}{12}$

Name________________________________

Angle Sum/Difference Identities

Date________________ Period____

Use the angle sum identity to find the exact value of each.

1) $\cos 105°$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

2) $\sin 195°$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

3) $\cos 195°$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

4) $\cos 165°$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

5) $\cos 285°$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

6) $\cos 255°$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

7) $\sin 105°$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

8) $\sin 285°$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

9) $\cos 75°$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

10) $\sin 255°$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

Use the angle difference identity to find the exact value of each.

11) $\cos 75°$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

12) $\cos -15°$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

13) $\tan 75°$

$2 + \sqrt{3}$

14) $\cos 15°$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

15) $\tan -105°$

$2 + \sqrt{3}$

16) $\sin 105°$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

17) $\tan 15°$

$2 - \sqrt{3}$

18) $\sin 15°$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

19) $\tan -15°$

$\sqrt{3} - 2$

20) $\sin -75°$

$\frac{-\sqrt{6} - \sqrt{2}}{4}$

Use the angle sum or difference identity to find the exact value of each.

21) $\sin -105°$

$\frac{-\sqrt{6} - \sqrt{2}}{4}$

22) $\cos 195°$

$\frac{-\sqrt{6} - \sqrt{2}}{4}$

23) $\cos \frac{7\pi}{12}$

$\frac{\sqrt{2} - \sqrt{6}}{4}$

24) $\tan \frac{13\pi}{12}$

$2 - \sqrt{3}$

25) $\sin \frac{\pi}{12}$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

26) $\cos -\frac{7\pi}{12}$

$\frac{\sqrt{2} - \sqrt{6}}{4}$

Sum and Difference Identities

Find the exact value of each trigonometric expression.

1. cos 75°

SOLUTION:

Write 75° as the sum or difference of angle measures with cosines that you know.

$$\begin{aligned}\cos 75^\circ &= \cos(45^\circ + 30^\circ)\\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ\\ &= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

2. sin (–210°)

SOLUTION:

Write –210° as the sum or difference of angle measures with sines that you know.

$$\begin{aligned}\sin(-210^\circ) &= \sin(60^\circ - 270^\circ)\\ &= \sin 60^\circ \cos 270^\circ - \cos 60^\circ \sin 270^\circ\\ &= \frac{\sqrt{3}}{2}(0) - \frac{1}{2}(-1)\\ &= \frac{1}{2}\end{aligned}$$

3. $\sin \frac{11\pi}{12}$

SOLUTION:

Write $\frac{11\pi}{12}$ as the sum or difference of angle measures with sines that you know.

$$\begin{aligned}\sin\frac{11\pi}{12} &= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)\\ &= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

4. $\cos \frac{17\pi}{12}$

SOLUTION:

Write $\frac{17\pi}{12}$ as the sum or difference of angle measures with cosines that you know.

$$\begin{aligned}\cos\frac{17\pi}{12} &= \cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)\\ &= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}\\ &= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

Sum and Difference Identities

5. $\tan \frac{23\pi}{12}$

SOLUTION:

Write $\frac{23\pi}{12}$ as the sum or difference of angle measures with tangents that you know.

$$\begin{aligned}\tan\frac{23\pi}{12} &= \tan\left(\frac{7\pi}{6}+\frac{3\pi}{4}\right)\\ &= \frac{\tan\frac{7\pi}{6}+\tan\frac{3\pi}{4}}{1-\tan\frac{7\pi}{6}\tan\frac{3\pi}{4}}\\ &= \frac{\frac{\sqrt{3}}{3}+(-1)}{1-\frac{\sqrt{3}}{3}(-1)}\\ &= \frac{\frac{\sqrt{3}-3}{3}}{\frac{3+\sqrt{3}}{3}}\\ &= \frac{\sqrt{3}-3}{3+\sqrt{3}}\\ &= \frac{-3+\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}}\\ &= \frac{-9+6\sqrt{3}-3}{9-3}\\ &= \frac{-12+6\sqrt{3}}{6}\\ &= -2+\sqrt{3}\end{aligned}$$

6. $\tan \frac{\pi}{12}$

SOLUTION:

Write $\frac{\pi}{12}$ as the sum or difference of angle measures with cosines that you know.

$$\begin{aligned}\tan\frac{\pi}{12} &= \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\\ &= \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}\\ &= \frac{\sqrt{3}-1}{1+\sqrt{3}(1)}\\ &= \frac{-1+\sqrt{3}}{1+\sqrt{3}}\\ &= \frac{-1+\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}\\ &= \frac{-1+2\sqrt{3}-3}{1-3}\\ &= \frac{-4+2\sqrt{3}}{-2}\\ &= 2-\sqrt{3}\end{aligned}$$

7. **VOLTAGE** Analysis of the voltage in a hairdryer involves terms of the form sin (nwt – 90°), where n is a positive integer, w is the frequency of the voltage, and t is time. Use an identity to simplify this expression.

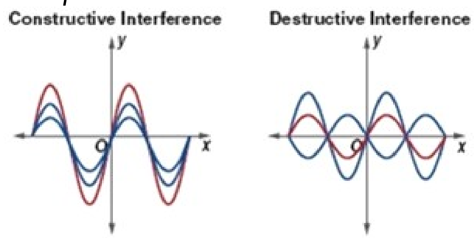
SOLUTION:

sin (nwt – 90°)

$$\begin{aligned}\sin(nwt-90^\circ) &= \sin nwt\cos 90^\circ-\cos nwt\sin 90^\circ\\ &= \sin nwt(0)-\cos nwt(1)\\ &= -\cos nwt\end{aligned}$$

Sum and Difference Identities

8. **BROADCASTING** When the sum of the amplitudes of two waves is greater than that of the component waves, the result is *constructive interference*. When the component waves combine to have a smaller amplitude, *destructive interference* occurs.



Consider two signals modeled by $y = 10 \sin (2t + 30°)$ and $y = 10 \sin (2t + 210°)$.

a. Find the sum of the two functions.

b. What type of interference results when the signals modeled by the two equations are combined?

SOLUTION:

a. $10 \sin (2t + 30°) + 10 \sin (2t + 210°)$.

$$\begin{aligned} &10\sin(2t+30°)+10\sin(2t+210°)\\ &=10(\sin 2t\cos 30°+\cos 2t\sin 30°)+10(\sin 2t\cos 210°+\cos 2t\sin 210°)\\ &=10\left[\sin 2t\left(\frac{\sqrt{3}}{2}\right)+\cos 2t\left(\frac{1}{2}\right)\right]+10\left[\sin 2t\left(-\frac{\sqrt{3}}{2}\right)+\cos 2t\left(-\frac{1}{2}\right)\right]\\ &=10\left[\sin 2t\left(\frac{\sqrt{3}}{2}\right)+\cos 2t\left(\frac{1}{2}\right)\right]-10\left[\sin 2t\left(\frac{\sqrt{3}}{2}\right)+\cos 2t\left(\frac{1}{2}\right)\right]\\ &=0 \end{aligned}$$

b. The combination of the two functions is zero, and the amplitude of the constant function is 0. The amplitude of each original function is 10. Since $0 < 10$, their sum can be characterized as producing destructive interference.

9. **WEATHER** The monthly high temperatures for Minneapolis can be modeled by $f(x) = 31.65\sin\left(\frac{\pi}{6}x - 2.09\right) + 52.35$, where x represents the months in which January = 1, February = 2, and so on. The monthly low temperatures for Minneapolis can be modeled by $g(x) = 31.65\sin\left(\frac{\pi}{6}x - 2.09\right) + 32.95$.

a. Write a new function $h(x)$ by adding the two functions and dividing the result by 2.

b. What does the function you wrote in part **a** represent?

SOLUTION:

a.

$$\begin{aligned} h(x) &= \frac{f(x)+g(x)}{2}\\ &=\frac{31.65\sin\left(\frac{\pi}{6}x-2.09\right)+52.35+31.65\sin\left(\frac{\pi}{6}x-2.09\right)+32.95}{2}\\ &=\frac{63.3\sin\left(\frac{\pi}{6}x-2.09\right)+85.3}{2}\\ &=31.65\sin\left(\frac{\pi}{6}x-2.09\right)+42.65 \end{aligned}$$

b. Since $f(x)$ represents the high temperatures and $g(x)$ the low temperatures for Minneapolis for each month x, the sum of these two functions divided by 2 represents the average of the high and low temperatures for each month.

Sum and Difference Identities

10. **TECHNOLOGY** A blind mobility aid uses the same idea as a bat's sonar to enable people who are visually impaired to detect objects around them. The sound wave emitted by the device for a certain patient can be modeled by $b = 30(\sin 195°)t$, where t is time in seconds and b is air pressure in pascals.
a. Rewrite the formula in terms of the difference of two angle measures.
b. What is the pressure after 1 second?

SOLUTION:

a.

$$b = 30(\sin 195°)t$$
$$b = 30\sin(240° - 45°)t$$

b.

$$\begin{aligned} b &= 30\sin(240° - 45°)t \\ &= 30\sin(240° - 45°)(1) \\ &= 30[(\sin 240°)(\cos 45°) - (\cos 240°)(\sin 45°)] \\ &= 30\left[\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\right] \\ &= 30\left[\left(-\frac{\sqrt{6}}{4}\right) + \frac{\sqrt{2}}{4}\right] \\ &= 30\left(\frac{\sqrt{2} - \sqrt{6}}{4}\right) \\ &= \frac{30\sqrt{2} - 30\sqrt{6}}{4} \\ &\approx -7.8 \end{aligned}$$

The pressure after 1 second is –7.8 pascals.

Find the exact value of each expression.

11. $\dfrac{\tan 43° - \tan 13°}{1 + \tan 43° \tan 13°}$

SOLUTION:

$$\begin{aligned} \frac{\tan 43° - \tan 13°}{1 + \tan 43° \tan 13°} &= \tan(43° - 13°) \\ &= \tan 30° \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

12. $\cos \frac{5\pi}{12} \cos \frac{\pi}{4} + \sin \frac{5\pi}{12} \sin \frac{\pi}{4}$

SOLUTION:

$$\begin{aligned} \cos\frac{5\pi}{12}\cos\frac{\pi}{4} + \sin\frac{5\pi}{12}\sin\frac{\pi}{4} &= \cos\left(\frac{5\pi}{12} - \frac{\pi}{4}\right) \\ &= \cos\frac{2\pi}{12} \\ &= \cos\frac{\pi}{6} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

13. $\sin 15° \cos 75° + \cos 15° \sin 75°$

SOLUTION:

$$\begin{aligned} \sin 15° \cos 75° + \cos 15° \sin 75° &= \sin(15° + 75°) \\ &= \sin 90° \\ &= 1 \end{aligned}$$

14. $\sin \frac{\pi}{3} \cos \frac{\pi}{12} - \cos \frac{\pi}{3} \sin \frac{\pi}{12}$

SOLUTION:

$$\begin{aligned} \sin\frac{\pi}{3}\cos\frac{\pi}{12} - \cos\frac{\pi}{3}\sin\frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{12}\right) \\ &= \sin\frac{3\pi}{12} \\ &= \sin\frac{\pi}{4} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

15. $\cos 40° \cos 20° - \sin 40° \sin 20°$

SOLUTION:

$$\begin{aligned} \cos 40° \cos 20° - \sin 40° \sin 20° &= \cos(40° + 20°) \\ &= \cos 60° \\ &= \frac{1}{2} \end{aligned}$$

16. $\dfrac{\tan 48° + \tan 12°}{1 - \tan 48° \tan 12°}$

SOLUTION:

$$\begin{aligned} \frac{\tan 48° + \tan 12°}{1 - \tan 48° \tan 12°} &= \tan(48° + 12°) \\ &= \tan 60° \\ &= \sqrt{3} \end{aligned}$$

Sum and Difference Identities

Simplify each expression.

17. $\dfrac{\tan 2\theta - \tan\theta}{1+\tan 2\theta \tan\theta}$

SOLUTION:

$$\frac{\tan 2\theta - \tan\theta}{1+\tan 2\theta \tan\theta} = \tan(2\theta - \theta) = \tan\theta$$

18. $\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

SOLUTION:

$$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x = \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

19. $\sin 3y \cos y + \cos 3y \sin y$

SOLUTION:

$$\sin 3y\cos y + \cos 3y \sin y = \sin(3y + y) = \sin 4y$$

20. $\cos 2x \sin x - \sin 2x \cos x$

SOLUTION:

$$\begin{aligned}&\cos 2x \sin x - \sin 2x \cos x\\ &= -(\sin 2x \cos x - \cos 2x \sin x)\\ &= -\sin(2x - x)\\ &= -\sin x\end{aligned}$$

21. $\cos x \cos 2x + \sin x \sin 2x$

SOLUTION:

$$\cos x\cos 2x + \sin x \sin 2x = \cos(x - 2x) = \cos(-x) = \cos x$$

22. $\dfrac{\tan 5\theta + \tan\theta}{\tan 5\theta \tan\theta - 1}$

SOLUTION:

$$\begin{aligned}\frac{\tan 5\theta + \tan\theta}{\tan 5\theta \tan\theta - 1} &= \frac{\tan 5\theta + \tan\theta}{-(1 - \tan 5\theta\tan\theta)}\\ &= -\frac{\tan 5\theta + \tan\theta}{1 - \tan 5\theta \tan\theta}\\ &= -\tan(5\theta + \theta)\\ &= -\tan 6\theta\end{aligned}$$

23. **SCIENCE** An electric circuit contains a capacitor, an inductor, and a resistor. The voltage drop across the inductor is given by $V_L = IwL\cos\left(wt + \frac{\pi}{2}\right)$, where I is the peak current, w is the frequency, L is the inductance, and t is time. Use the cosine sum identity to express V_L as a function of $\sin wt$.

SOLUTION:

$$\begin{aligned}V_L &= IwL\cos\left(wt + \frac{\pi}{2}\right)\\ V_L &= IwL\left(\cos wt \cos\frac{\pi}{2} - \sin wt \sin\frac{\pi}{2}\right)\\ V_L &= IwL[\cos wt(0) - \sin wt(1)]\\ V_L &= IwL(-\sin wt)\\ V_L &= -IwL\sin wt\end{aligned}$$

Sum and Difference Identities

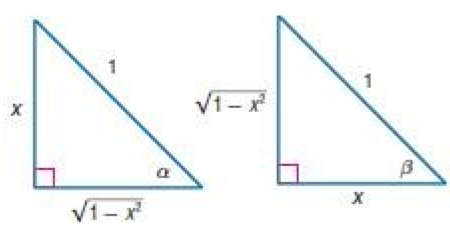
Write each trigonometric expression as an algebraic expression.

24. sin (arcsin x + arccos x)

SOLUTION:

$$\sin(\arcsin x + \arccos x)$$
$$= \sin(\arcsin x)\cos(\arccos x) + \cos(\arcsin x)\sin(\arccos x)$$
$$= x(x) + \cos(\arcsin x)\sin(\arccos x)$$
$$= x^2 + \cos(\arcsin x)\sin(\arccos x)$$

If we let $\alpha = \arcsin x$ and $\beta = \arccos x$, then $\sin\alpha = x$ and $\cos\beta = x$. Sketch one right triangle with an acute angle α and another with an acute angle β. Label the sides such that $\sin\alpha = x$ and $\cos\beta = x$. Then use the Pythagorean Theorem to express the length of each third side.



Using these triangles, we find that $\cos(\arcsin x) = \cos\alpha$ or $\sqrt{1-x^2}$ and $\sin(\arccos x) = \sin\beta$ or $\sqrt{1-x^2}$. Substitute these values in $x^2 + \cos(\arcsin x)\sin(\arccos x)$ and simplify.

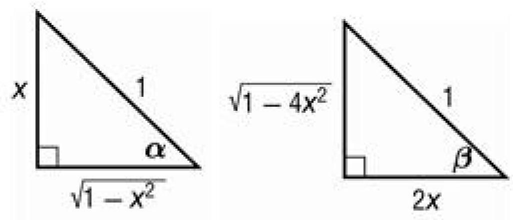
$$\sin(\arcsin x + \arccos x)$$
$$= x^2 + \cos(\arcsin x)\sin(\arccos x)$$
$$= x^2 + \left(\sqrt{1-x^2}\right)\left(\sqrt{1-x^2}\right)$$
$$= x^2 + \left(1-x^2\right)$$
$$= 1$$

25. $\cos\left(\sin^{-1} x + \cos^{-1} 2x\right)$

SOLUTION:

$$\cos\left(\sin^{-1}x + \cos^{-1}2x\right)$$
$$= \cos\left(\sin^{-1}x\right)\cos\left(\cos^{-1}2x\right) - \sin\left(\sin^{-1}x\right)\sin\left(\cos^{-1}2x\right)$$
$$= 2x\cos\left(\sin^{-1}x\right) - x\sin\left(\cos^{-1}2x\right)$$

If we let $\alpha = \sin^{-1} x$ and $\beta = \cos^{-1} 2x$, then $\sin\alpha = x$ Sketch one right triangle with an acute angle α and a acute angle β. Label the sides such that $\sin\alpha = x$ an Then use the Pythagorean Theorem to express the le side.



Using these triangles, we find that $\cos\left(\sin^{-1} x\right) = \cos\alpha$ or $\sqrt{1-x^2}$ and $\sin\left(\cos^{-1} 2x\right) = s$ Substitute these values in $2x\cos\left(\sin^{-1} x\right) - x\sin\left(\cos^{-}\right.$ simplify.

$$\cos\left(\sin^{-1}x + \cos^{-1}2x\right)$$
$$= 2x\cos\left(\sin^{-1}x\right) - x\sin\left(\cos^{-1}2x\right)$$
$$= 2x\left(\sqrt{1-x^2}\right) - x\left(\sqrt{1-4x^2}\right)$$

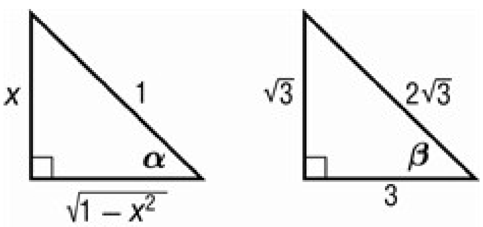
Sum and Difference Identities

26. $\cos\left(\sin^{-1}x - \tan^{-1}\frac{\sqrt{3}}{3}\right)$

SOLUTION:

$$\cos\left(\sin^{-1}x - \tan^{-1}\frac{\sqrt{3}}{3}\right)$$
$$= \cos(\sin^{-1}x)\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) + \sin(\sin^{-1}x)\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$$
$$= \cos(\sin^{-1}x)\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) + x\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$$

If we let $\alpha = \sin^{-1}x$ and $\beta = \tan^{-1}\frac{\sqrt{3}}{3}$, then $\sin\alpha = x$ and $\tan\beta = \frac{\sqrt{3}}{3}$. Sketch one right triangle with an acute angle α and another with an acute angle β. Label the sides such that $\sin\alpha = x$ and $\tan\beta = \frac{\sqrt{3}}{3}$. Then use the Pythagorean Theorem to express the length of each third side.



Using these triangles, we find that $\cos(\sin^{-1}x) = \cos\alpha$ or $\sqrt{1-x^2}$, $\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = \cos\beta$ or $\frac{\sqrt{3}}{2}$, and $\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = \sin\beta$ or $\frac{1}{2}$. Substitute these values in $\cos(\sin^{-1}x)\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) + x\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ and simplify.

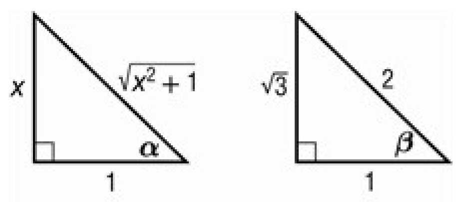
$$\cos(\sin^{-1}x)\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) + x\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$$
$$= \sqrt{1-x^2}\left(\frac{\sqrt{3}}{2}\right) + x\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3-3x^2}}{2} + \frac{x}{2}$$
$$= \frac{\sqrt{3-3x^2}+x}{2}$$

27. $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2} - \tan^{-1}x\right)$

SOLUTION:

$$\sin\left(\sin^{-1}\frac{\sqrt{3}}{2} - \tan^{-1}x\right)$$
$$= \sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\cos(\tan^{-1}x) - \cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\sin(\tan^{-1}x)$$
$$= \frac{\sqrt{3}}{2}\cos(\tan^{-1}x) - \cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\sin(\tan^{-1}x)$$

If we let $\alpha = \tan^{-1}x$ and $\beta = \sin^{-1}\frac{\sqrt{3}}{2}$, then $\tan\alpha = x$ and $\sin\beta = \frac{\sqrt{3}}{2}$. Sketch one right triangle with an acute angle α and another with an acute angle β. Label the sides such that $\tan\alpha = x$ and $\sin\beta = \frac{\sqrt{3}}{2}$. Then use the Pythagorean Theorem to express the length of each third side.



Using these triangles, we find that $\cos(\tan^{-1}x) = \cos\alpha$ or $\frac{\sqrt{x^2+1}}{x^2+1}$, $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \cos\beta$ or $\frac{1}{2}$, and $\sin(\tan^{-1}x) = \sin\alpha$ or $\frac{x\sqrt{x^2+1}}{x^2+1}$. Substitute these values in $\frac{\sqrt{3}}{2}\cos(\tan^{-1}x) - \cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\sin(\tan^{-1}x)$ and simplify.

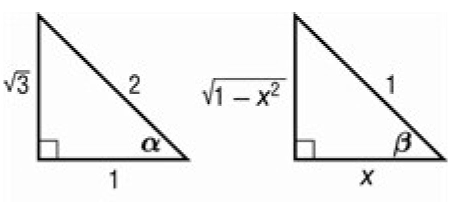
$$\frac{\sqrt{3}}{2}\cos\left(\tan^{-1}x\right)-\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\sin\left(\tan^{-1}x\right)$$
$$=\left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{x^2+1}}{x^2+1}-\frac{1}{2}\left(\frac{x\sqrt{x^2+1}}{x^2+1}\right)$$
$$=\frac{\sqrt{3}\sqrt{x^2+1}-x\sqrt{x^2+1}}{2\left(x^2+1\right)}$$
$$=\frac{(\sqrt{3}-x)\sqrt{x^2+1}}{2x^2+2}$$

28. $\cos\left(\arctan\sqrt{3}-\arccos x\right)$

SOLUTION:

$$\cos\left(\arctan\sqrt{3}-\arccos x\right)$$
$$=\cos\left(\arctan\sqrt{3}\right)\cos(\arccos x)+\sin\left(\arctan\sqrt{3}\right)\sin(\arccos x)$$
$$=\cos\left(\arctan\sqrt{3}\right)x+\sin\left(\arctan\sqrt{3}\right)\sin(\arccos x)$$

If we let $\alpha=\arctan\sqrt{3}$ and $\beta=\arccos x$, then $\tan\alpha=\sqrt{3}$ and $\cos\beta=x$. Sketch one right triangle with an acute angle α and another with an acute angle β. Label the sides such that $\tan\alpha=\sqrt{3}$ and $\cos\beta=x$. Then use the Pythagorean Theorem to express the length of each third side.



Using these triangles, we find that

$\cos\left(\arctan\sqrt{3}\right)=\cos\alpha$ or $\frac{1}{2}$,

$\sin\left(\arctan\sqrt{3}\right)=\sin\alpha$ or $\frac{\sqrt{3}}{2}$, and

$\sin\left(\arccos x\right)=\sin\beta$ or $\sqrt{1-x^2}$. Substitute these values and simplify.

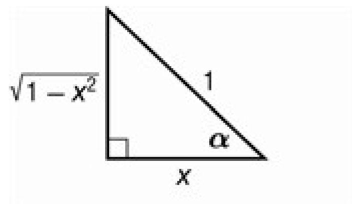
$$\cos\left(\arctan\sqrt{3}\right)x+\sin\left(\arctan\sqrt{3}\right)\sin(\arccos x)$$
$$=\frac{1}{2}x+\frac{\sqrt{3}}{2}\left(\sqrt{1-x^2}\right)$$
$$=\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}$$
$$=\frac{x+\sqrt{3-3x^2}}{2}$$

29. $\tan(\cos^{-1}x+\tan^{-1}x)$

SOLUTION:

$$\tan(\cos^{-1}x+\tan^{-1}x)=\frac{\tan\left(\cos^{-1}x\right)+\tan\left(\tan^{-1}x\right)}{1-\left[\tan\left(\cos^{-1}x\right)\right]\left[\tan\left(\tan^{-1}x\right)\right]}$$
$$=\frac{\tan\left(\cos^{-1}x\right)+x}{1-x\tan\left(\cos^{-1}x\right)}$$

If we let $\alpha=\cos^{-1}x$, then $\cos\alpha=x$. Sketch a right triangle with an acute angle α. Label the sides such that $\cos\alpha=x$. Then use the Pythagorean Theorem to express the length of the third side.



Using this triangle, we find that

$\tan\left(\cos^{-1}x\right)=\tan\alpha$ or $\frac{\sqrt{1-x^2}}{x}$. Substitute this value in $\frac{\tan\left(\cos^{-1}x\right)+x}{1-x\tan\left(\cos^{-1}x\right)}$ and simplify.

$$\frac{\tan\left(\cos^{-1}x\right)+x}{1-x\tan\left(\cos^{-1}x\right)}$$

$$=\frac{\frac{\sqrt{1-x^2}}{x}+x}{1-x\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$=\frac{\frac{x^2+\sqrt{1-x^2}}{x}}{1-\sqrt{1-x^2}}$$

$$=\frac{x^2+\sqrt{1-x^2}}{x}\cdot\frac{1}{1-\sqrt{1-x^2}}$$

$$=\frac{x^2+\sqrt{1-x^2}}{x-x\sqrt{1-x^2}}$$

$$=\frac{x^2+\sqrt{1-x^2}}{x-x\sqrt{1-x^2}}\cdot\frac{x+x\sqrt{1-x^2}}{x+x\sqrt{1-x^2}}$$

$$=\frac{x^3+x^3\sqrt{1-x^2}+x\sqrt{1-x^2}+x\left(1-x^2\right)}{x^2-x^2\left(1-x^2\right)}$$

$$=\frac{x^3+x^3\sqrt{1-x^2}+x\sqrt{1-x^2}+x-x^3}{x^2-x^2+x^4}$$

$$=\frac{x+\left(x^3+x\right)\sqrt{1-x^2}}{x^4}$$

$$=\frac{x+x\left(x^2+1\right)\sqrt{1-x^2}}{x^4}$$

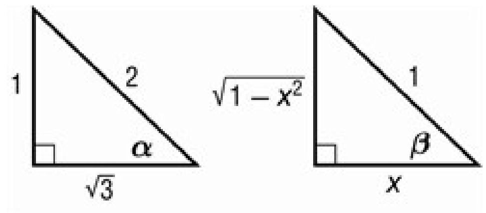
$$=\frac{1+\left(x^2+1\right)\sqrt{1-x^2}}{x^3}$$

30. $\tan\left(\sin^{-1}\frac{1}{2}-\cos^{-1}x\right)$

SOLUTION:

$$\tan\left(\sin^{-1}\frac{1}{2}-\cos^{-1}x\right)=\frac{\tan\left(\sin^{-1}\frac{1}{2}\right)-\tan\left(\cos^{-1}x\right)}{1+\left[\tan\left(\sin^{-1}\frac{1}{2}\right)\right]\left[\tan\left(\cos^{-1}x\right)\right]}$$

If we let $\alpha=\sin^{-1}\frac{1}{2}$ and $\beta=\cos^{-1}x$ then $\sin\alpha=\frac{1}{2}$ and $\cos\beta=x$. Sketch one right triangle with an acute angle α and another with an acute angle β. Label the sides such that $\sin\alpha=\frac{1}{2}$ and $\cos\beta=x$. Then use the Pythagorean Theorem to express the length of each third side.



Using these triangles, we find that $\tan\left(\sin^{-1}\frac{1}{2}\right)=\tan\alpha$ or $\frac{\sqrt{3}}{3}$ and $\tan\left(\cos^{-1}x\right)=\tan\beta$ or $\frac{\sqrt{1-x^2}}{x}$. Substitute these values and simplify.

$$\frac{\tan\left(\sin^{-1}\frac{1}{2}\right)-\tan\left(\cos^{-1}x\right)}{1+\left[\tan\left(\sin^{-1}\frac{1}{2}\right)\right]\left[\tan\left(\cos^{-1}x\right)\right]}$$

$$=\frac{\frac{\sqrt{3}}{3}-\frac{\sqrt{1-x^2}}{x}}{1+\left(\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$=\frac{\frac{\sqrt{3}x-3\sqrt{1-x^2}}{3x}}{\frac{3x+\sqrt{3}\sqrt{1-x^2}}{3x}}$$

$$=\frac{\sqrt{3}x-3\sqrt{1-x^2}}{3x+\sqrt{3}\sqrt{1-x^2}}$$

$$=\frac{\sqrt{3}x-3\sqrt{1-x^2}}{3x+\sqrt{3-3x^2}}$$

$$=\frac{\sqrt{3}x-3\sqrt{1-x^2}}{3x+\sqrt{3-3x^2}}\cdot\frac{3x-\sqrt{3}\sqrt{1-x^2}}{3x-\sqrt{3-3x^2}}$$

$$=\frac{3\sqrt{3}x^2-9x\sqrt{1-x^2}-3x\sqrt{1-x^2}+3\sqrt{3}\left(1-x^2\right)}{9x^2-\left(3-3x^2\right)}$$

$$=\frac{3\sqrt{3}x^2-12x\sqrt{1-x^2}+3\sqrt{3}-3\sqrt{3}x^2}{9x^2-3+3x^2}$$

$$=\frac{3\sqrt{3}-12x\sqrt{1-x^2}}{12x^2-3}$$

$$=\frac{\sqrt{3}-4x\sqrt{1-x^2}}{4x^2-1}$$

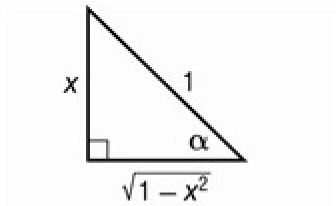
Sum and Difference Identities

31. $\tan\left(\sin^{-1}x + \frac{\pi}{4}\right)$

SOLUTION:

$$\tan\left(\sin^{-1}x + \frac{\pi}{4}\right) = \frac{\tan(\sin^{-1}x) + \tan\frac{\pi}{4}}{1 - \tan(\sin^{-1}x)\tan\frac{\pi}{4}} = \frac{\tan(\sin^{-1}x) + 1}{1 - \tan(\sin^{-1}x)}$$

If we let $\alpha = \sin^{-1}x$, then $\sin\alpha = x$. Sketch a right triangle with an acute angle α. Label the sides such that $\sin\alpha = x$. Then use the Pythagorean Theorem to express the length of the third side.



Using this triangle, we find that

$\tan(\sin^{-1}x) = \tan\alpha$ or $\frac{x\sqrt{1-x^2}}{1-x^2}$. Substitute this value in $\frac{\tan(\sin^{-1}x)+1}{1-\tan(\sin^{-1}x)}$ and simplify.

$$\begin{aligned}
\frac{\tan(\sin^{-1}x)+1}{1-\tan(\sin^{-1}x)} &= \frac{\frac{x\sqrt{1-x^2}}{1-x^2}+1}{1-\frac{x\sqrt{1-x^2}}{1-x^2}} \\
&= \frac{\frac{x\sqrt{1-x^2}+1-x^2}{1-x^2}}{\frac{1-x^2-x\sqrt{1-x^2}}{1-x^2}} \\
&= \frac{x\sqrt{1-x^2}+1-x^2}{1-x^2-x\sqrt{1-x^2}} \\
&= \frac{(1-x^2)+x\sqrt{1-x^2}}{(1-x^2)-x\sqrt{1-x^2}} \cdot \frac{(1-x^2)+x\sqrt{1-x^2}}{(1-x^2)+x\sqrt{1-x^2}} \\
&= \frac{(1-x^2)^2+2x(1-x^2)\sqrt{1-x^2}+x^2(1-x^2)}{(1-x^2)^2-x^2(1-x^2)} \\
&= \frac{1-2x^2+x^4+2x(1-x^2)\sqrt{1-x^2}+x^2-x^4}{1-2x^2+x^4-x^2+x^4} \\
&= \frac{-x^2+1+(2x-2x^3)\sqrt{1-x^2}}{2x^4-3x^2+1}
\end{aligned}$$

Verify each cofunction identity using one or more difference identities.

32. $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

SOLUTION:

$\tan\left(\frac{\pi}{2}-x\right) = \frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$	Quotient Identity
$= \frac{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x}$	Trig Difference Identities
$= \frac{1(\cos x) - 0(\sin x)}{0(\cos x) + 1(\sin x)}$	$\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$
$= \frac{\cos x}{\sin x}$	Simplify.
$= \cot x$	Quotient Identity

33. $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

SOLUTION:

$\sec\left(\frac{\pi}{2}-x\right) = \frac{1}{\cos\left(\frac{\pi}{2}-x\right)}$	Quotient Identity
$= \frac{1}{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x}$	Cosine Difference Identity
$= \frac{1}{0(\cos x) + 1(\sin x)}$	$\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$
$= \frac{1}{\sin x}$	Simplify.
$= \csc x$	Quotient Identity

34. $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

SOLUTION:

$\cot\left(\frac{\pi}{2}-x\right) = \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$	Quotient Identity
$= \frac{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x}{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x}$	Trig Difference Identities
$= \frac{0(\cos x) + 1(\sin x)}{1(\cos x) - 0(\sin x)}$	$\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$
$= \frac{\sin x}{\cos x}$	Simplify.
$= \tan x$	Quotient Identity

Verify each reduction identity.

35. $\cos(\pi - \theta) = -\cos\theta$

SOLUTION:

$\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta$	Cosine Difference Identity
$= -1(\cos\theta) + 0(\sin\theta)$	$\cos\pi = -1$ and $\sin\pi = 0$
$= -\cos\theta$	Simplify.

Sum and Difference Identities

36. $\cos(2\pi + \theta) = \cos\theta$

SOLUTION:

$\cos(2\pi+\theta) = \cos 2\pi\cos\theta - \sin 2\pi\sin\theta$	Cosine Sum Identity
$= 1(\cos\theta) - 0(\sin\theta)$	$\sin 2\pi = 0$ $\cos 2\pi = 1$
$= 1(\cos\theta) - 0$	Multiply.
$= \cos\theta$	Simplify.

37. $\sin(\pi - \theta) = \sin\theta$

SOLUTION:

$\sin(\pi-\theta) = \sin\pi\cos\theta - \cos\pi\sin\theta$	Sine Difference Identity
$= 0(\cos\theta) - (-1)(\sin\theta)$	$\sin\pi = 0$ and $\cos\pi = -1$
$= \sin\theta$	Simplify.

38. $\sin(90° + \theta) = \cos\theta$

SOLUTION:

$\sin(90°+\theta) = \sin 90°\cos\theta + \cos 90°\sin\theta$	Sine Sum Identity
$= 1(\cos\theta) + 0(\sin\theta)$	$\sin 90° = 1$ and $\cos 90° = 0$
$= \cos\theta$	Simplify.

39. $\cos(270° - \theta) = -\sin\theta$

SOLUTION:

$\cos(270°-\theta) = \cos 270°\cos\theta + \sin 270°\sin\theta$	Cosine Difference Identity
$= 0(\cos\theta) + (-1)(\sin\theta)$	$\cos 270° = 0$ and $\sin 270° = -1$
$= -\sin\theta$	Simplify.

Find the solution to each expression on the interval $[0, 2\pi)$.

40. $\cos\left(\frac{\pi}{2}+x\right) - \sin\left(\frac{\pi}{2}+x\right) = 0$

SOLUTION:

$$\cos\left(\frac{\pi}{2}+x\right) - \sin\left(\frac{\pi}{2}+x\right) = 0$$
$$\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x - \left[\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x\right] = 0$$
$$(0)\cos x - (1)\sin x - (1)\cos x - (0)\sin x = 0$$
$$-\sin x - \cos x = 0$$
$$\sin x + \cos x = 0$$
$$\sin x = -\cos x$$

On the interval $[0, 2\pi)$, $\sin x = -\cos x$ when $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$.

41. $\cos(\pi + x) + \cos(\pi + x) = 1$

SOLUTION:

$$\cos(\pi+x) + \cos(\pi+x) = 1$$
$$2\cos(\pi+x) = 1$$
$$\cos(\pi+x) = \frac{1}{2}$$
$$\cos\pi\cos x - \sin\pi\sin x = \frac{1}{2}$$
$$(-1)\cos x - (0)\sin x = \frac{1}{2}$$
$$-\cos x = \frac{1}{2}$$
$$\cos x = -\frac{1}{2}$$

On the interval $[0, 2\pi)$, $\cos x = -\frac{1}{2}$ when $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

42. $\cos\left(\frac{\pi}{6}+x\right) + \sin\left(\frac{\pi}{3}+x\right) = 0$

SOLUTION:

$$\cos\left(\frac{\pi}{6}+x\right) + \sin\left(\frac{\pi}{3}+x\right) = 0$$
$$\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x + \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x = 0$$
$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 0$$
$$\sqrt{3}\cos x = 0$$
$$\cos x = 0$$

On the interval $[0, 2\pi)$, $\cos x = 0$ when $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

Sum and Difference Identities

43. $\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=\frac{1}{2}$

SOLUTION:

$$\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=\frac{1}{2}$$
$$\sin\frac{\pi}{6}\cos x+\cos\frac{\pi}{6}\sin x+\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x=\frac{1}{2}$$
$$\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\frac{1}{2}$$
$$\cos x=\frac{1}{2}$$

On the interval [0, 2π), $\cos x=\frac{1}{2}$ when $x=\frac{\pi}{3}$ and $x=\frac{5\pi}{3}$.

44. $\sin\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{3\pi}{2}+x\right)=-2$

SOLUTION:

$$\sin\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{3\pi}{2}+x\right)=-2$$
$$2\sin\left(\frac{3\pi}{2}+x\right)=-2$$
$$\sin\left(\frac{3\pi}{2}+x\right)=-1$$
$$\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x=-1$$
$$(-1)\cos x+(0)\sin x=-1$$
$$-\cos x=-1$$
$$\cos x=1$$

On the interval [0, 2π), $\cos x=1$ when $x=0$.

45. $\tan(\pi+x)+\tan(\pi+x)=2$

SOLUTION:

$$\tan(\pi+x)+\tan(\pi+x)=2$$
$$2\tan(\pi+x)=2$$
$$\tan(\pi+x)=1$$
$$\frac{\tan\pi+\tan x}{1-\tan\pi\tan x}=1$$
$$\frac{0+\tan x}{1-(0)\tan x}=1$$
$$\frac{\tan x}{1}=1$$
$$\tan x=1$$

On the interval [0, 2π), $\tan x=1$ when $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

Verify each identity.

46. $\tan x-\tan y=\frac{\sin(x-y)}{\cos x\cos y}$

SOLUTION:

$$\frac{\sin(x-y)}{\cos x\cos y}$$
$$=\frac{\sin x\cos y-\cos x\sin y}{\cos x\cos y}\quad\text{Difference Identity}$$
$$=\frac{\sin x\cos y}{\cos x\cos y}-\frac{\cos x\sin y}{\cos x\cos y}\quad\text{Split numerator.}$$
$$=\frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}\quad\text{Simplify.}$$
$$=\tan x-\tan y\quad\text{Quotient Identity}$$

47. $\cot\alpha-\tan\beta=\frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}$

SOLUTION:

$$\frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}$$
$$=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta}\quad\text{Sine Sum Identity}$$
$$=\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}\quad\text{Split fraction.}$$
$$=\frac{\cos\alpha}{\sin\alpha}-\frac{\sin\beta}{\cos\beta}\quad\text{Divide out common factors.}$$
$$=\cot\alpha-\tan\beta\quad\text{Quotient Identities}$$

Sum and Difference Identities

48. $\dfrac{(\tan u - \tan v)}{(\tan u + \tan v)} = \dfrac{\sin(u - v)}{\sin(u + v)}$

SOLUTION:

$$\frac{\tan u - \tan v}{\tan u + \tan v}$$

$$= \frac{\frac{\sin u}{\cos u} - \frac{\sin v}{\cos v}}{\frac{\sin u}{\cos u} + \frac{\sin v}{\cos v}} \quad \text{Quotient Identity}$$

$$= \frac{\frac{\sin u \cos v}{\cos u \cos v} - \frac{\sin v \cos u}{\cos u \cos v}}{\frac{\sin u \cos v}{\cos u \cos v} + \frac{\sin v \cos u}{\cos u \cos v}} \quad \text{Rewrite fractions.}$$

$$= \frac{\sin u \cos v - \sin v \cos u}{\sin u \cos v + \sin v \cos u} \quad \text{Combine fractions.}$$

$$= \frac{\sin(u - v)}{\sin(u + v)} \quad \text{Sine +/- Identities}$$

49. $2 \sin a \cos b = \sin(a + b) + \sin(a - b)$

SOLUTION:

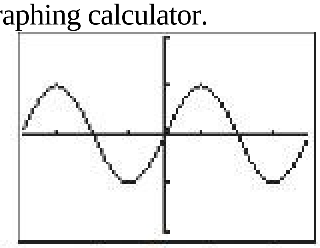
$$\sin(a + b) + \sin(a - b)$$
$$= \sin a \cos b + \cos a \sin b + \sin a \cos b - \cos a \sin b$$
$$= 2 \sin a \cos b$$

GRAPHING CALCULATOR Graph each function and make a conjecture based on the graph. Verify your conjecture algebraically.

50. $y = \frac{1}{2}[\sin(x + 2\pi) + \sin(x - 2\pi)]$

SOLUTION:

Graph $y = \frac{1}{2}[\sin(x + 2\pi) + \sin(x - 2\pi)]$ using a graphing calculator.



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

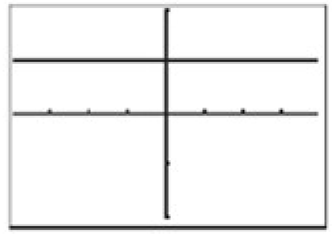
The function appears to be equivalent to $y = \sin x$.

$$\frac{1}{2}[\sin(x + 2\pi) + \sin(x - 2\pi)]$$
$$= \frac{1}{2}[(\sin x \cos 2\pi + \cos x \sin 2\pi) + (\sin x \cos 2\pi - \cos x \sin 2\pi)] \quad \text{Sine +/- Identities}$$
$$= \frac{1}{2}(2 \sin x \cos 2\pi) \quad \text{Combine like terms.}$$
$$= \sin x \cos 2\pi \quad \text{Multiply.}$$
$$= \sin x(1) \quad \cos 2\pi = 1,\ \sin 2\pi = 0$$
$$= \sin x \quad \text{Multiply.}$$

51. $y = \cos^2\left(x + \frac{\pi}{4}\right) + \cos^2\left(x - \frac{\pi}{4}\right)$

SOLUTION:

Graph $y = \cos^2\left(x + \frac{\pi}{4}\right) + \cos^2\left(x - \frac{\pi}{4}\right)$ using a graphing calculator.

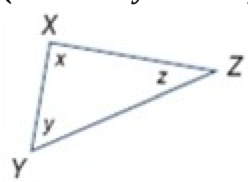


$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

The function appears to be equivalent to $y = 1$.

$$\cos^2\left(x + \frac{\pi}{4}\right) + \cos^2\left(x - \frac{\pi}{4}\right)$$
$$= \left[\cos\left(x + \frac{\pi}{4}\right)\right]^2 + \left[\cos\left(x - \frac{\pi}{4}\right)\right]^2$$
$$= \left(\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}\right)^2 + \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right)^2$$
$$= \left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)^2 + \left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)^2$$
$$= \left(\frac{1}{2}\cos^2 x - \sin x \cos x + \frac{1}{2}\sin^2 x\right) + \left(\frac{1}{2}\cos^2 x + \sin x \cos x + \frac{1}{2}\sin^2 x\right)$$
$$= \cos^2 x + \sin^2 x$$
$$= 1$$

PROOF Consider ΔXYZ. Prove each identity. (*Hint*: $x + y + z = \pi$)



52. $\cos(x + y) = -\cos z$

SOLUTION:

$$\cos(x + y) = \cos(\pi - z) \quad x + y = \pi - z$$
$$= \cos\pi \cos z + \sin\pi \sin z \quad \text{Cosine Difference Identity}$$
$$= -1(\cos z) + 0 \quad \cos\pi = -1 \text{ and } \sin\pi = 0$$
$$= -\cos z \quad \text{Simplify.}$$

53. $\sin z = \sin x \cos y + \cos x \sin y$

SOLUTION:

$$\sin z = \sin[\pi - (x + y)] \quad z = \pi - (x + y)$$
$$= \sin\pi \cos(x + y) - \cos\pi \sin(x + y) \quad \text{Sine of Difference Identity}$$
$$= 0 \cdot \cos(x + y) - [(-1)\sin(x + y)] \quad \sin\pi = 0 \text{ and } \cos\pi = -1$$
$$= \sin(x + y) \quad \text{Simplify.}$$
$$= \sin x \cos y + \cos x \sin y \quad \text{Sine Sum Identity}$$

Sum and Difference Identities

54. $\tan x + \tan y + \tan z = \tan x \tan y \tan z$

SOLUTION:

$$
\begin{aligned}
&\tan x + \tan y + \tan z \\
&= \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} + \frac{\sin z}{\cos z} \\
&= \frac{\sin x \cos y \cos z}{\cos x \cos y \cos z} + \frac{\sin y \cos x \cos z}{\cos x \cos y \cos z} + \frac{\sin z \cos x \cos y}{\cos x \cos y \cos z} \\
&= \frac{\cos z(\sin x \cos y + \sin y \cos x)}{\cos x \cos y \cos z} + \frac{\sin z \cos x \cos y}{\cos x \cos y \cos z} \\
&= \frac{\cos z \sin(x+y)}{\cos x \cos y \cos z} + \frac{\sin z \cos x \cos y}{\cos x \cos y \cos z} \\
&= \frac{\cos z \sin(x+y)}{\cos x \cos y \cos z} + \frac{\sin z(\cos x \cos y - \sin x \sin y + \sin x \sin y)}{\cos x \cos y \cos z} \\
&= \frac{\cos z \sin(x+y)}{\cos x \cos y \cos z} + \frac{\sin z[\cos(x+y) + \sin x \sin y]}{\cos x \cos y \cos z} \\
&= \frac{\cos z \sin(\pi - z)}{\cos x \cos y \cos z} + \frac{\sin z[\cos(\pi - z) + \sin x \sin y]}{\cos x \cos y \cos z} \\
&= \frac{\cos z(\sin \pi \cos z - \cos \pi \sin z)}{\cos x \cos y \cos z} + \frac{\sin z[(\cos \pi \cos z + \sin \pi \sin z) + \sin x \sin y]}{\cos x \cos y \cos z} \\
&= \frac{\cos z[0 \cdot \cos z - (-1)\sin z]}{\cos x \cos y \cos z} + \frac{\sin z\{[(-1)\cos z + 0 \cdot \sin z] + \sin x \sin y\}}{\cos x \cos y \cos z} \\
&= \frac{\cos z \sin z - \cos z \sin z + \sin x \sin y \sin z}{\cos x \cos y \cos z} \\
&= \frac{\sin z \sin y \sin z}{\cos x \cos y \cos z} \\
&= \tan x \tan y \tan z
\end{aligned}
$$

55. **CALCULUS** The difference quotient is given by $\dfrac{f(x+h) - f(x)}{h}$.

a. Let $f(x) = \sin x$. Write and expand an expression for the difference quotient.

b. Set your answer from part **a** equal to y. Use a graphing calculator to graph the function for the following values of h: 2, 1, 0.1, and 0.01.

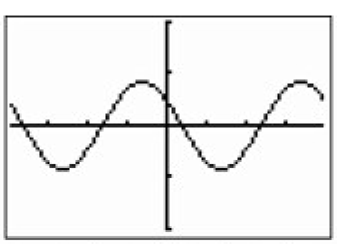
c. What function does the graph in part **b** resemble as h approaches zero?

SOLUTION:

a.

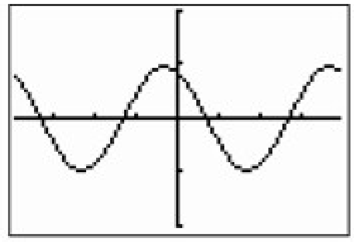
$$
\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\sin(x+h) - \sin x}{h} \\
&= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}
\end{aligned}
$$

b. $y = \dfrac{\sin x \cos(2) + \cos x \sin(2) - \sin x}{2}$



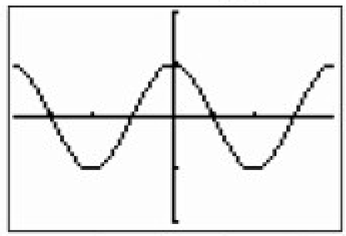
$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

$y = \sin x \cos(1) + \cos x \sin(1) - \sin x$



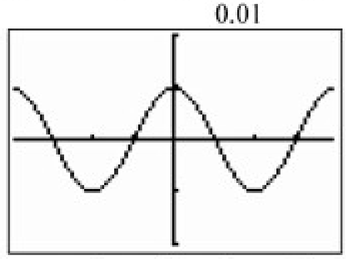
$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

$y = \dfrac{\sin x \cos(0.1) + \cos x \sin(0.1) - \sin x}{0.1}$



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

$y = \dfrac{\sin x \cos(00.1) + \cos x \sin(0.01) - \sin x}{0.01}$

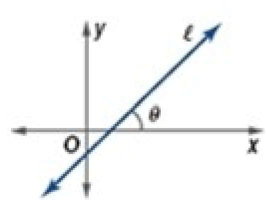


$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

c. $\cos x$

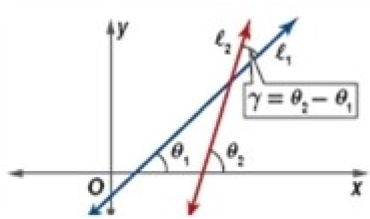
56. **ANGLE OF INCLINATION** The *angle of inclination* θ of a line is the angle formed between the positive x-axis and the line, where $0° < \theta < 180°$.

a. Prove that the slope m of line ℓ is given by $m = \tan \theta$.



b. Consider lines ℓ_1 and ℓ_2 below with slopes m_1 and m_2, respectively. Derive a formula for the angle γ formed by the two lines.

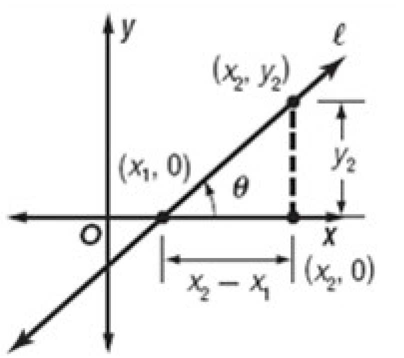
Sum and Difference Identities



c. Use the formula you found in part **b** to find the angle formed by $y = \frac{\sqrt{3}}{3}x$ and $y = x$.

SOLUTION:

a.



$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$	Definition of slope
$= \frac{y_2 - 0}{x_2 - x_1}$	$(x_2, y_2) = (x_2, 0)$
$= \frac{y_2}{x_2 - x_1}$	Simplify.
$= \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$	In the diagram, y_2 is the side opposite and $(x_2 - x_1)$ is the side adjacent.
$= \tan\theta$	Definition of tangent ratio

b. From part **a**, we know that the slope of a line is equivalent to the tangent of its angle of inclination. Therefore $\tan \theta_1 = m_1$, $\tan \theta_2 = m_2$. We are given that the angle formed by the intersection of the two lines γ is equivalent to $\theta_2 - \theta_1$. Use this information to derive a formula for γ.

$\tan\gamma = \tan(\theta_2 - \theta_1)$	$\gamma = \theta_2 - \theta_1$
$\tan\gamma = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$	Tangent Difference Identity
$\tan\gamma = \frac{m_2 - m_1}{1 + m_1 m_2}$	$\tan\theta_2 = m_2$ and $\tan\theta_1 = m_1$
$\gamma = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$	Definition of inverse tangent

c. Let $y = \frac{\sqrt{3}}{3}x$ be line 1 and $y = x$ be line 2. The slope of line 1 is $\frac{\sqrt{3}}{3}$ and of line 2 is 1. Therefore, $m_1 = \frac{\sqrt{3}}{3}$ and $m_2 = 1$.

$\gamma = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$	Formula for γ
$= \tan^{-1}\left(\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}(1)}\right)$	$m_2 = 1$ and $m_1 = \frac{\sqrt{3}}{3}$
$= \tan^{-1}\left(\frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}}\right)$	Simplify.
$= \tan^{-1}\left(\frac{3 - \sqrt{3}}{3 + \sqrt{3}}\right)$	Divide.
$= 15°$	Use a calculator set in Degree Mode.

PROOF Verify each identity.

57. $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

SOLUTION:

$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$	Quotient Identity
$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$	Sine and Cosine Sum Identities
$= \frac{\frac{1}{\cos\alpha\cos\beta}(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{\frac{1}{\cos\alpha\cos\beta}(\cos\alpha\cos\beta - \sin\alpha\sin\beta)}$	Multiply numerator and denominator by $\frac{1}{\cos\alpha\cos\beta}$.
$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$	Distributive Property
$= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$	Divide out common factors.
$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	Quotient Identity

58. $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

SOLUTION:

$\tan(\alpha - \beta)$	
$= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$	Quotient Identity
$= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$	Trig Difference Identities
$= \frac{\frac{1}{\cos\alpha\cos\beta}(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\frac{1}{\cos\alpha\cos\beta}(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$	Multiply numerator and denominator by $\frac{1}{\cos\alpha\cos\beta}$.
$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$	Distributive Property
$= \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}}{1 + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$	Divide out common factors.
$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$	Quotient Identity

Sum and Difference Identities

59. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

SOLUTION:

$\sin(\alpha+\beta) = \cos[90° - (\alpha+\beta)]$	Cofunction Identity
$= \cos[90° - \alpha - \beta]$	Distributive Property
$= \cos[(90° - \alpha) - \beta]$	Associative Property
$= \cos(90° - \alpha)\cos\beta + \sin(90° - \alpha)\sin\beta$	Cosine Difference Identity
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$	Cofunction Identities

60. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

SOLUTION:

$\sin(\alpha-\beta) = \sin[\alpha + (-\beta)]$	Rewrite subtraction as addition.
$= \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$	Sine Sum Identity
$= \sin\alpha\cos\beta + \cos\alpha(-\sin\beta)$	Odd-Even Identities
$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$	Multiply

61. **REASONING** Use the sum identity for sine to derive an identity for $\sin(x + y + z)$ in terms of sines and cosines.

SOLUTION:

$$\begin{aligned}&\sin(x+y+z)\\&=\sin[(x+y)+z]\\&=\sin(x+y)\cos z+\cos(x+y)\sin z\\&=(\sin x\cos y+\cos x\sin y)\cos z+(\cos x\cos y-\sin x\sin y)\sin z\\&=\sin x\cos y\cos z+\cos x\sin y\cos z+\cos x\cos y\sin z-\sin x\sin y\sin z\end{aligned}$$

CHALLENGE If $\sin x = -\frac{2}{3}$ and $\cos y = \frac{1}{3}$, find each of the following if x is in Quadrant IV and y is in Quadrant I.

62. $\cos(x + y)$

SOLUTION:

If $\sin x = -\frac{2}{3}$ and x is in Quadrant IV, then $\cos x = \frac{\sqrt{3^2 - (-2)^2}}{3}$ or $\frac{\sqrt{5}}{3}$. If $\cos y = \frac{1}{3}$ and y is in Quadrant I, then $\sin y = \frac{\sqrt{3^2 - 1^2}}{3}$ or $\frac{2\sqrt{2}}{3}$.

$$\begin{aligned}\cos(x+y)&=\cos x\cos y-\sin x\sin y\\&=\left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{3}\right)-\left(-\frac{2}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)\\&=\frac{\sqrt{5}}{9}-\left(-\frac{4\sqrt{2}}{9}\right)\\&=\frac{\sqrt{5}+4\sqrt{2}}{9}\end{aligned}$$

63. $\sin(x - y)$

SOLUTION:

If $\sin x = -\frac{2}{3}$ and x is in Quadrant IV, then $\cos x = \frac{\sqrt{3^2 - (-2)^2}}{3}$ or $\frac{\sqrt{5}}{3}$. If $\cos y = \frac{1}{3}$ and y is in Quadrant I, then $\sin y = \frac{\sqrt{3^2 - 1^2}}{3}$ or $\frac{2\sqrt{2}}{3}$.

$$\begin{aligned}\sin(x-y)&=\sin x\cos y-\cos x\sin y\\&=\left(-\frac{2}{3}\right)\left(\frac{1}{3}\right)-\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)\\&=-\frac{2}{9}-\frac{2\sqrt{10}}{9}\\&=\frac{-2-2\sqrt{10}}{9}\end{aligned}$$

Sum and Difference Identities

64. tan $(x + y)$

SOLUTION:

If $\sin x = -\frac{2}{3}$ and x is in Quadrant IV, then tan x = $-\frac{2}{\sqrt{3^2-2^2}}$ or $-\frac{2}{\sqrt{5}}$. If $\cos y = \frac{1}{3}$ and y is in Quadrant I, then tan y = $\frac{\sqrt{3^2-1^2}}{1}$ or $2\sqrt{2}$.

$$\begin{aligned}
\tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\
&= \frac{-\frac{2}{\sqrt{5}} + 2\sqrt{2}}{1 - \left(-\frac{2}{\sqrt{5}}\right)\left(2\sqrt{2}\right)} \\
&= \frac{\frac{-2+2\sqrt{10}}{\sqrt{5}}}{1 + \left(\frac{4\sqrt{2}}{\sqrt{5}}\right)} \\
&= \frac{\frac{-2+2\sqrt{10}}{\sqrt{5}}}{\frac{\sqrt{5}+4\sqrt{2}}{\sqrt{5}}} \\
&= \frac{-2+2\sqrt{10}}{\sqrt{5}+4\sqrt{2}} \\
&= \frac{-2+2\sqrt{10}}{\sqrt{5}+4\sqrt{2}} \cdot \frac{\sqrt{5}-4\sqrt{2}}{\sqrt{5}-4\sqrt{2}} \\
&= \frac{-2\sqrt{5}+8\sqrt{2}+2\sqrt{50}-8\sqrt{20}}{5-16(2)} \\
&= \frac{-2\sqrt{5}+8\sqrt{2}+2\sqrt{25\cdot 2}-8\sqrt{4\cdot 5}}{-27} \\
&= \frac{-2\sqrt{5}+8\sqrt{2}+10\sqrt{2}-16\sqrt{5}}{-27} \\
&= \frac{-18\sqrt{5}+18\sqrt{2}}{-27} \\
&= \frac{-9\left(2\sqrt{5}-2\sqrt{2}\right)}{-27} \\
&= \frac{2\sqrt{5}-2\sqrt{2}}{3}
\end{aligned}$$

65. **REASONING** Consider sin 3x cos 2x = cos 3x sin 2x.

a. Find the solutions of the equation over [0, 2π) algebraically.

b. Support your answer graphically.

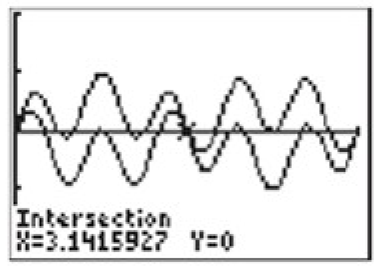
SOLUTION:

a.

$\sin 3x \cos 2x = \cos 3x \sin 2x$	Original equation
$\sin 3x \cos 2x - \cos 3x \sin 2x = 0$	Subtract $\cos 3x \sin 2x$.
$\sin(3x - 2x) = 0$	Sine Difference Identity
$\sin x = 0$	Combine like terms.

On the interval [0, 2π), $\sin x = 0$ when $x = 0$ or $x = \pi$.

b. Graph **Y1** = $\sin 3x \cos 2x$ and **Y2** = $\cos 3x \sin 2x$. The intersection of these two graphs on [0, 2π) is at x = 0 and x = π.



[−0, 2π] scl: $\frac{\pi}{2}$ by [−2, 2] scl: 1

PROOF Prove each difference quotient identity.

66. $\frac{\sin(x+h) - \sin x}{h} = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x \frac{\sin h}{h}$

SOLUTION:

$\frac{\sin(x+h) - \sin x}{h}$	
$= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	Sine Sum Identity
$= \frac{(\sin x \cos h - \sin x) + \cos x \sin h}{h}$	Com. and Assoc. Prop.
$= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$	Factor.
$= \frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}$	Sum of two fractions
$= \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)$	Factor.

Sum and Difference Identities

67. $\dfrac{\cos(x+h)-\cos x}{h}=\cos x\left(\dfrac{\cos h-1}{h}\right)-\sin x\dfrac{\sin h}{h}$

SOLUTION:

$\dfrac{\cos(x+h)-\cos x}{h}$

$=\dfrac{\cos x\cos h-\sin x\sin h-\cos x}{h}$ Cosine Sum Identity

$=\dfrac{(\cos x\cos h-\cos x)-\sin x\sin h}{h}$ Com. and Asso. Prop..

$=\dfrac{\cos x(\cos h-1)-\sin x\sin h}{h}$ Factor.

$=\dfrac{\cos x(\cos h-1)}{h}-\dfrac{\sin x\sin h}{h}$ Sum of two fractions.

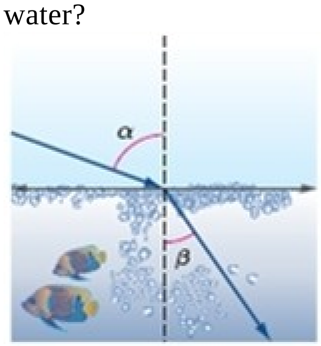
$=\cos x\left(\dfrac{\cos h-1}{h}\right)-\sin x\left(\dfrac{\sin h}{h}\right)$ Factor.

68. ***Writing in Math*** Can a tangent sum or difference identity be used to solve any tangent reduction formula? Explain your reasoning.

SOLUTION:

No; sample answer: A tangent sum or difference identity can be used to solve a tangent reduction formula as long as the angle is not a multiple of $\dfrac{\pi}{2}$ radians, since $\tan\left(\dfrac{\pi}{2}\right)$ does not exist.

69. **PHYSICS** According to Snell's law, the angle at which light enters water α is related to the angle at which light travels in water β by $\sin\alpha = 1.33\sin\beta$. At what angle does a beam of light enter the water if the beam travels at an angle of $23°$ through the water?



SOLUTION:

$\sin\alpha = 1.33\sin\beta$

$\sin\alpha = 1.33\sin 23°$

$\alpha = \sin^{-1}(1.33\sin 23°)$

$\alpha \approx 31.3°$

Verify each identity.

70. $\dfrac{\cos\theta}{1-\sin^2\theta}=\sec\theta$

SOLUTION:

$\dfrac{\cos\theta}{1-\sin^2\theta}=\dfrac{\cos\theta}{\cos^2\theta}$ Pythagorean Identity

$=\dfrac{1}{\cos\theta}$ Simplify.

$=\sec\theta$ Reciprocal Identity

71. $\dfrac{\sec\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}=\cot\theta$

SOLUTION:

$\dfrac{\sec\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{1}{\sin\theta\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$ Reciprocal Identity

$=\dfrac{1}{\sin\theta\cos\theta}-\dfrac{\sin^2\theta}{\sin\theta\cos\theta}$ Rewrite fractions using a common denominator.

$=\dfrac{\cos^2\theta}{\sin\theta\cos\theta}$ Pythagorean Identity

$=\dfrac{\cos\theta}{\sin\theta}$ Simplify .

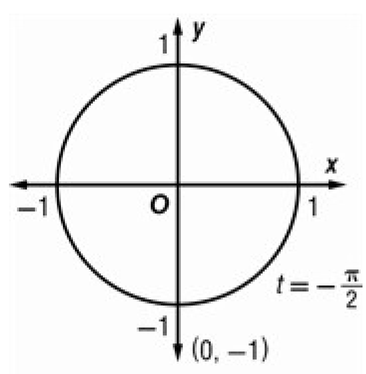
$=\cot\theta$

Find the exact value of each expression, if it exists.

72. $\sin^{-1}(-1)$

SOLUTION:

Find a point on the unit circle on the interval $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ with a y-coordinate of –1.



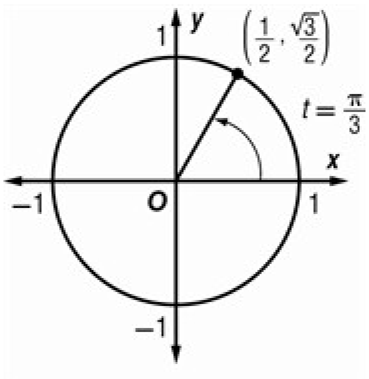
When $t=-\dfrac{\pi}{2}$, $\sin t = -1$. Therefore, $\sin^{-1}(-1) = -\dfrac{\pi}{2}$.

Sum and Difference Identities

73. $\tan^{-1}\sqrt{3}$

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\frac{y}{x} = \sqrt{3}$.

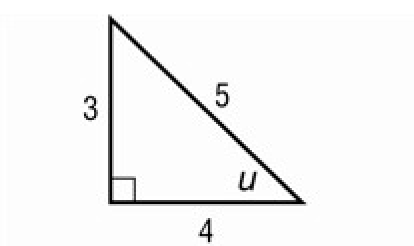


When $t = \frac{\pi}{3}$, $\tan t = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$. Therefore, arctan ($\sqrt{3}$) = $\frac{\pi}{3}$.

74. $\tan\left(\arcsin\frac{3}{5}\right)$

SOLUTION:

If we let $u = \arcsin\frac{3}{5}$, then $\sin u = \frac{3}{5}$. Sketch a right triangle with an acute angle *u*. Label the sides such that $\sin u = \frac{3}{5}$. Then use the Pythagorean Theorem to express the length of the third side.



Using this triangle, we find that $\tan\left(\arcsin\frac{3}{5}\right) = \tan u$ or $\frac{3}{4}$.

75. **MONEY** Suppose you deposit a principal amount of *P* dollars in a bank account that pays compound interest. If the annual interest rate is *r* (expressed as a decimal) and the bank makes interest payments *n* times every year, the amount of money *A* you would have after *t* years is given by $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$.

a. If the principal, interest rate, and number of interest payments are known, what type of function is $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$? Explain your reasoning.

b. Write an equation giving the amount of money you would have after *t* years if you deposit $1000 into an account paying 4% annual interest compounded quarterly (four times per year).

c. Find the account balance after 20 years.

SOLUTION:

a. Since the values are assumed to be known, let p = 1000, n = 12, and r = 6%. Substitute these values into the equation.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$
$$A(t) = 1000\left(1+\frac{0.06}{12}\right)^{12t}$$
$$A(t) = 1000(1.005)^{12t}$$

After simplifying, we determine that $A(t)$ is exponential. Regardless of the values that we substitute, the coefficient and the base of the exponent will be fixed while the exponent will be variable.

b. We are given a value of 1000 for *P* as well as a rate *r* of 0.04. Interest is compounded quarterly, so we can assume 4 payments per year. Substitute these values into the equation.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$
$$A(t) = 1000\left(1+\frac{0.04}{4}\right)^{4t}$$
$$A(t) = 1000(1.01)^{4t}$$

c. To find the balance after 20 years, substitute 20 for *t*.

$A(t) = 1000(1.01)^{4t}$

$A(20) = 1000(1.01)^{4(20)}$

$A(20) \approx 2216.72$

The account balance will be $2216.72 after 20 years.

List all possible rational zeros of each function. Then determine which, if any, are zeros.

76. $p(x) = x^4 + x^3 - 11x - 5x + 30$

SOLUTION:

Because the leading coefficient is 1, the possible rational zeros are the integer factors of the constant term 30. Therefore, the possible rational zeros of *g* are ±1, ±2, ±3, ±5, ±6, ±10, ±15, and ±30.

By using synthetic division, it can be determined that 2 is a rational zero.

2	1	1	−11	−5	30
		2	6	−10	−30
	1	3	−5	−15	0

By using synthetic division on the depressed polynomial, it can be determined that –3 is a rational zero.

−3	1	3	−5	−15
		−3	0	15
	1	0	−5	0

Because $(x - 2)$ and $(x + 3)$ are factors of $g(x)$, we can use the final quotient to write a factored form of $g(x)$ as $g(x) = (x - 2)(x + 3)(x^2 - 5)$. Because the factor $(x^2 - 5)$ yields no rational zeros, the rational zeros of *p* are –3 and 2.

77. $d(x) = 2x^4 - x^3 - 6x^2 + 5x - 1$

SOLUTION:

The leading coefficient is 6 and the constant term is −1. The possible rational zeros are $\pm\frac{1}{6}$, $\pm\frac{1}{3}$, $\pm\frac{1}{2}$, and ±1.

By using synthetic division, it can be determined that $\frac{1}{2}$ is the only rational zero.

$\frac{1}{2}$	2	−1	−6	5	−1
		1	0	−3	1
	2	0	−6	2	0

The depressed polynomial $2x^3 - 3x + 1$ yields no rational zeros. Therefore, the rational zero of *d* is $\frac{1}{2}$.

78. $f(x) = x^3 - 2x^2 - 5x - 6$

SOLUTION:

Because the leading coefficient is 1, the possible rational zeros are the integer factors of the constant term −6. Therefore, the possible rational zeros of *g* are ±1, ±2, ±3, and ±6.

Using synthetic division, it does not appear that the polynomial has any rational zeros.

Sum and Difference Identities

79. **SAT/ACT** There are 16 green marbles, 2 red marbles, and 6 yellow marbles in a jar. How many yellow marbles need to be added to the jar in order to double the probability of selecting a yellow marble?

A 4
B 6
C 8
D 12
E 16

SOLUTION:

First find the probability of selecting a yellow.

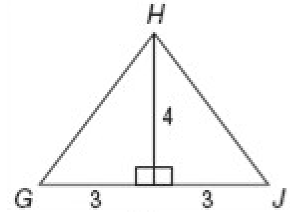
$P(Y) = \frac{6}{24} = \frac{1}{4}$.

For this probability to double, it would have to be $2\left(\frac{1}{4}\right)$ or $\frac{1}{2}$. Let y be the number of yellow marbles you would need to add to the jar to make the probability equal $\frac{1}{2}$.

$$\frac{6+y}{24+y} = \frac{1}{2}$$
$$y + 24 = 2(6+y)$$
$$y + 24 = 12 + 2y$$
$$12 = y$$

A total of 12 yellow marbles will need to be added to the jar to double the probability of selecting a yellow.

80. **REVIEW** Refer to the figure below. Which equation could be used to find $m\angle G$?



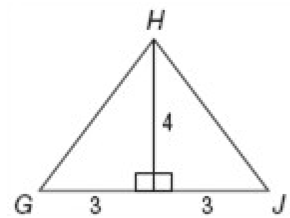
F $\sin G = \frac{3}{4}$

G $\cos G = \frac{3}{4}$

H $\cot G = \frac{3}{4}$

J $\tan G = \frac{3}{4}$

SOLUTION:



Using the Pythagorean Theorem, we know that *GH* = 5. Any of the following trigonometric ratios could be used to find *G*.

$$\sin G = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$$
$$\cos G = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$$
$$\tan G = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$
$$\csc G = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$
$$\sec G = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$
$$\cot G = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

Only choice H has a correct ratio given.

Sum and Difference Identities

81. Find the exact value of sin θ.

A $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

B $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

C $\dfrac{2+\sqrt{3}}{4}$

D $\dfrac{2-\sqrt{3}}{4}$

SOLUTION:

Since $\theta = 60° + 45°$, use the Sine Sum Identity.

$$\begin{aligned}\sin\theta &= \sin(60° + 45°)\\ &= \sin 60° \cos 45° + \cos 60° \sin 45°\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

A is the correct choice.

82. **REVIEW** Which of the following is equivalent to $\dfrac{\cos\theta(\cot^2\theta + 1)}{\csc\theta}$?

F tan θ
G cot θ
H sec θ
J csc θ

SOLUTION:

$\dfrac{\cos\theta(\cot^2\theta+1)}{\csc\theta}$

$= \dfrac{\cos\theta(\csc^2\theta)}{\csc\theta}$	Pythagorean Identity
$= \cos\theta\csc\theta$	Simplify.
$= \cos\theta\dfrac{1}{\sin\theta}$	Reciprocal Identity
$= \dfrac{\cos\theta}{\sin\theta}$	Multiply.
$= \cot\theta$	Quotient Identity

Santa Color by Number

Use the information below to color in this picture to find out what's happening.

1 = white $\frac{1}{2}$ = dark blue $\frac{\sqrt{3}}{2}$ = red $\frac{\sqrt{2}}{2}$ = orange 0 = yellow $\frac{\sqrt{3}}{3}$ = green -1 = brown $\sqrt{3}$ = pink