**Sum and Difference Identities**

Classify each statement as true or false, then explain your reasoning.

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1) sin(90°) = sin(90°) + sin(0°) |  | 3) sin(30°) + sin(60°) = sin(90°) |
|  |  |  |  |
|  | 2) cos(90°) = cos(90°) + cos(0°) |  | 4) tan(45°) + tan(45°) = tan(90°) |
|  |  |  |  |

From these examples, recognize that the distributive property does NOT apply

when using the trigonometric functions.

…So what rules *do* apply?

|  |  |
| --- | --- |
| **Sum and Difference Identities** | |
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|  |  |
| --- | --- |
| **A. Demonstrate** | Demonstrate that each identity works by substituting the given values for *A* and *B*. |

Sample: , when  and 

Write the identity: 

Substitute: 

Evaluate: 

Simplify: 

0 = 0 ✓

|  |  |
| --- | --- |
| 5) | , when  and |
|  |  |

**Sum and Difference Identities**

|  |  |
| --- | --- |
| 6) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 7) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 8) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 9) | , when  and |
|  |  |

**Sum and Difference Identities**

|  |  |
| --- | --- |
| **B. Find** | Find the exact values for the trigonometric functions at 105° and 15° by writing the angles as a sum or difference of other special values (such as 60° and 45°). |

Sample: 



|  |  |
| --- | --- |
| 10) |  |
|  |  |

|  |  |
| --- | --- |
| 11) |  |
|  |  |

|  |  |
| --- | --- |
| 12) |  |
|  |  |

|  |  |
| --- | --- |
| 13) |  |
|  |  |

**Sum and Difference Identities**

|  |  |
| --- | --- |
| 14) |  |
|  |  |

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| --- | --- |
| **C. Find** | Use the sum and difference identities to evaluate functions of other rotation angles. |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *R* and *S* are rotation angles (between 0° and 90°) whose terminal sides intersect the unit circle at  and , respectively.   |  |  |  |  | | --- | --- | --- | --- | | 15) | Write down the sine, cosine and tangent values for each angle. | | | |  | sin *R* = | cos *R* = | tan *R* = | |  | sin *S* = | cos *S* = | tan *S* = | |

|  |  |  |
| --- | --- | --- |
| 16) | With a calculator in degree mode, solve for the measures of *R* and *S*. Then use these values to estimate *R*+*S* and *R−S*. | |
|  | *R* = | *R*+*S* = |
|  | *S* = | *R−S* = |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 17) | Use the sum and difference identities to find the exact values for the trigonometric functions for *R*+*S* and *R−S*. | | | |
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| --- | --- |
| 18) | How can the answers to #16 be used to check the answers to #17 (using a calculator)? |
|  |  |

**Sum and Difference Identities**

|  |  |
| --- | --- |
| **D. Prove** | Use the sum and difference identities to verify (or prove) other identities. |

Sample: Verify that 

|  |  |  |
| --- | --- | --- |
|  |  | (Sum/Difference Identity) |
|  |  | (Evaluate/Substitute) |
|  |  | (Simplify) |

|  |  |
| --- | --- |
| 19) | Prove |
|  |  |

|  |  |
| --- | --- |
| 20) | Prove |
|  |  |

|  |  |  |
| --- | --- | --- |
| 21) | Prove | *Hint: Rewrite −x as* “0 – *x*” |
|  |  | |

|  |  |  |
| --- | --- | --- |
| 22) | Prove | *Hint: Rewrite −x as* “0 – *x*” |
|  |  | |

|  |  |  |
| --- | --- | --- |
| 23) | Prove | *Hint: Rewrite −x as* “0 – *x*” |
|  |  | |

**Sum and Difference Identities KEY**

Classify each statement as true or false, then explain your reasoning.

|  |  |  |  |
| --- | --- | --- | --- |
| TRUE | 1) sin(90°) = sin(90°) + sin(0°) | FALSE | 3) sin(30°) + sin(60°) = sin(90°) |
|  | Yes, 1 = 1 + 0 |  | No, ½ + /2 ≠ 1 |
| FALSE | 2) cos(90°) = cos(90°) + cos(0°) | FALSE | 4) tan(45°) + tan(45°) = tan(90°) |
|  | No, 0 ≠ 0 + 1 |  | No, Left side = 2, right side is undefined |

From these examples, recognize that the distributive property does NOT apply

when using the trigonometric functions.

…So what rules *do* apply?

|  |  |
| --- | --- |
| **Sum and Difference Identities** | |
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|  |  |
|  |

|  |  |
| --- | --- |
| **A. Demonstrate** | Demonstrate that each identity works by substituting the given values for *A* and *B*. |

Sample: , when  and 

Write the identity: 

Substitute: 

Evaluate: 

Simplify: 

0 = 0 ✓

|  |  |
| --- | --- |
| 5) | , when  and |
|  |  |

**Sum and Difference Identities KEY**

|  |  |
| --- | --- |
| 6) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 7) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 8) | , when  and |
|  |  |

|  |  |
| --- | --- |
| 9) | , when  and |
|  | (Both sides of the equation are undefined.) |

**Sum and Difference Identities KEY**

|  |  |
| --- | --- |
| **B. Find** | Find the exact values for the trigonometric functions at 105° and 15° by writing the angles as a sum or difference of other special values (such as 60° and 45°). |

Sample: 



|  |  |
| --- | --- |
| 10) |  |
|  |  |

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| --- | --- |
| 11) |  |
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|  |  |
| --- | --- |
| 12) |  |
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| --- | --- |
| 13) |  |
|  |  |

**Sum and Difference Identities KEY**

|  |  |
| --- | --- |
| 14) |  |
|  |  |

|  |  |
| --- | --- |
| **C. Find** | Use the sum and difference identities to evaluate functions of other rotation angles. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *R* and *S* are rotation angles (between 0° and 90°) whose terminal sides intersect the unit circle at  and , respectively.   |  |  |  |  | | --- | --- | --- | --- | | 15) | Write down the sine, cosine and tangent values for each angle. | | | |  | sin *R* = 24/25 | cos *R* = 7/25 | tan *R* = 24/7 | |  | sin *S* = 4/5 | cos *S* = 3/5 | tan *S* = 4/3 | |

|  |  |  |
| --- | --- | --- |
| 16) | With a calculator in degree mode, solve for the measures of *R* and *S*. Then use these values to estimate *R*+*S* and *R−S*. | |
|  | *R* = 73.74° | *R*+*S* = 126.87° |
|  | *S* = 53.13° | *R−S* = 20.61° |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 17) | Use the sum and difference identities to find the exact values for the trigonometric functions for *R*+*S* and *R−S*. | | | |
|  |  | sin*R* cos*S* − cos*R* sin*S* |  | sin*R* cos*S* + cos*R* sin*S* |
|  |  | cos*R* cos*S* + sin*R* sin*S* |  | cos*R* cos*S* − sin*R* sin*S* |

|  |  |
| --- | --- |
| 18) | How can the answers to #16 be used to check the answers to #17 (using a calculator)? |
|  | sin(126.87°) ≈ 0.8 = 4/5. cos(126.87°) ≈ -0.6 = -3/5.  sin(20.61°) ≈ 0.352 = 44/125. cos(20.61°) ≈ 0.936 = 117/125.  More exact values could be found using the store key for angles R and S, i.e. sin-1(24/25) ENTER 73.73979529 STO ALPHA R and do the same for S, then use sin(R + S). |

**Sum and Difference Identities KEY**

|  |  |
| --- | --- |
| **D. Prove** | Use the sum and difference identities to verify (or prove) other identities. |

Sample: Verify that 

|  |  |  |
| --- | --- | --- |
|  |  | (Sum/Difference Identity) |
|  |  | (Evaluate/Substitute) |
|  |  | (Simplify) |

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| --- | --- |
| 19) | Prove |
|  |  |

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