اوجد كلاً من النهايات الآتية :

1) $$\lim_{x \to 4} \frac{x^2 - 8\sqrt{x}}{4 - x} =$$

$$\lim_{x \to 4} \frac{x^2 - 8\sqrt{x}}{4 - x} \times \frac{x^2 + 8\sqrt{x}}{x^2 + 8\sqrt{x}} = \lim_{x \to 4} \frac{x^4 - 64x}{4 - x} \times \frac{1}{32}$$

$$\frac{1}{32} \lim_{x \to 4} \frac{x(x^3 - 64)}{4 - x} = \frac{1}{32} \lim_{x \to 4} \frac{x(x - 4)(x^2 + 4x + 16)}{4 - x} = -6$$

2) $$\lim_{x \to 1} \frac{\sqrt{3x + 1} - 2}{x - 1}$$

$$= \lim_{x \to 1} \frac{\sqrt{3x + 1} - 2}{x - 1} \times \frac{\sqrt{3x + 1} + 2}{\sqrt{3x + 1} + 2} = \lim_{x \to 1} \frac{3x + 1 - 4}{x - 1} \times \frac{1}{4} = \frac{1}{4} \lim_{x \to 1} \frac{3x - 3}{x - 1}$$

$$\frac{1}{4} \lim_{x \to 1} \frac{3(x - 1)}{x - 1} = \frac{3}{4}$$

3) $$\lim_{x \to 3^+} \frac{9 - x^2}{\sqrt{x^2 - 6x + 9}}$$

$$|x - 3| = \begin{cases} x - 3 & , x \geq 3 \\ -x + 3 & , x < 3 \end{cases}$$

$$= \lim_{x \to 3^+} \frac{9 - x^2}{\sqrt{x^2 - 6x + 9}} = \lim_{x \to 3^+} \frac{9 - x^2}{\sqrt{(x - 3)^2}} = \lim_{x \to 3^+} \frac{9 - x^2}{|x - 3|} = \lim_{x \to 3^+} \frac{(3 - x)(3 + x)}{(x - 3)}$$

$$\lim_{x \to 3^+} -(3 + x) = -6$$

4) $$\lim_{x \to 3} \frac{(x - 1)^4 - 16}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x - 1)^4 - 16}{x - 3} = \lim_{x \to 3} \frac{((x - 1)^2 - 4)((x - 1)^2 + 4)}{x - 3}$$

$$\lim_{x \to 3} \frac{(x - 1 - 2)(x - 1 + 2)((x - 1)^2 + 4)}{x - 3} = \lim_{x \to 3} (x + 1)((x - 1)^2 + 4) = 32$$

5) $$\lim_{x \to 4^+} \frac{x^2 + |4 - x| - 16}{x - 4}$$

$$|4 - x| = \begin{cases} x - 4 & , x \geq 4 \\ 4 - x & , x < 4 \end{cases}$$

$$\lim_{x \to 4^+} \frac{x^2 + |4 - x| - 16}{x - 4} = \lim_{x \to 4^+} \frac{x^2 + x - 4 - 16}{x - 4} = \lim_{x \to 4^+} \frac{x^2 + x - 20}{x - 4}$$

$$\lim_{x \to 4^+} \frac{(x + 5)(x - 4)}{x - 4} = \lim_{x \to 4^+} (x + 5) = 9$$

6) $\lim\limits_{x\to 2} \dfrac{(x^2-4)\sqrt{x+7}}{4-\sqrt{5x+6}}$

$$= \lim_{x\to 2} \frac{(x^2-4)\sqrt{x+7}}{4-\sqrt{5x+6}} \times \frac{4+\sqrt{5x+6}}{4+\sqrt{5x+6}} = \lim_{x\to 2} \frac{(x-2)(x+2)\sqrt{x+7}}{16-5x-6} \times 8$$

$$8\lim_{x\to 2} \frac{(x-2)(x+2)\sqrt{x+7}}{10-5x} = 8\lim_{x\to 2} \frac{\overset{-1}{\cancel{(x-2)}}(x+2)\sqrt{x+7}}{5\cancel{(2-x)}} = \frac{-96}{5}$$

7) $\lim\limits_{x\to 3} \left(\dfrac{x}{x-3} - \dfrac{18}{x^2-9}\right)$ توحيد المقامات

$$= \lim_{x\to 3} \frac{x(x+3)-18}{x^2-9} == \lim_{x\to 3} \frac{x^2+3x-18}{x^2-9} = \lim_{x\to 3} \frac{(x+6)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$$

$$\lim_{x\to 3} \frac{(x+6)}{(x+3)} = \frac{9}{6} = \frac{3}{2}$$

8) $\lim\limits_{x\to 3} \dfrac{1}{x-3}\left(\dfrac{1}{x^2} - \dfrac{1}{9}\right)$ توحيد المقامات

$$= \lim_{x\to 3} \frac{1}{x-3}\left(\frac{1}{x^2} - \frac{1}{9}\right) = \lim_{x\to 3} \frac{1}{x-3}\left(\frac{9-x^2}{9x^2}\right) = \lim_{x\to 3} \frac{1}{\cancel{x-3}}\left(\frac{\overset{-1}{\cancel{(3-x)}}(3+x)}{9x^2}\right)$$

$$\lim_{x\to 3} \frac{-(3+x)}{9x^2} = \frac{-6}{81} = \frac{-2}{27}$$

$$9)\quad \lim_{x\to 3}\frac{(x-1)^2-(5-x)^2}{x^2-3x}$$

$$=\lim_{x\to 3}\frac{x^2-2x+1-25+10x-x^2}{x(x-3)}=\lim_{x\to 3}\frac{8x-24}{x(x-3)}$$

$$\lim_{x\to 3}\frac{8(x-3)}{x(x-3)}=\frac{8}{3}$$

$$10)\quad \lim_{x\to 0}\frac{\sin 2x}{2x+\tan 3x}$$

بقسمة كل من البسط و المقام على $x$

$$\lim_{x\to 0}\frac{\frac{\sin 2x}{x}}{\frac{2x}{x}+\frac{\tan 3x}{x}}=\frac{\lim_{x\to 0}\frac{\sin 2x}{x}}{\lim_{x\to 0}2+\lim_{x\to 0}\frac{\tan 3x}{x}}=\frac{2}{2+3}=\frac{2}{5}$$

$$11)\quad \lim_{x\to 1}\frac{\sin(x^3-1)}{x-1}$$

بقسمة كل من البسط و المقام على $x^3-1$

$$\lim_{x\to 1}\frac{\frac{\sin(x^3-1)}{(x^3-1)}}{\frac{x-1}{(x^3-1)}}=\frac{\lim_{x\to 1}\frac{\sin(x^3-1)}{(x^3-1)}}{\lim_{x\to 1}\frac{x-1}{(x-1)(x^2+x+1)}}=\frac{\lim_{x\to 1}\frac{\sin(x^3-1)}{(x^3-1)}}{\lim_{x\to 1}\frac{1}{(x^2+x+1)}}$$

$$=\frac{\lim_{x\to 1}\frac{\sin(x^3-1)}{(x^3-1)}}{\frac{1}{3}}=3\lim_{x\to 1}\frac{\sin(x^3-1)}{(x^3-1)}=3\lim_{y\to 0}\frac{\sin y}{y}=3\times 1=3$$

نفرض $y=x^3-1$

$x\to 1\ ,\quad y\to 0$

12) $\lim_{x \to -3} \frac{|2x+3|-3}{x+3}$ نعيد تعريف $|2x+3| = \begin{cases} 2x+3 & , x \geq \frac{-3}{2} \\ -2x-3 & , x < \frac{-3}{2} \end{cases}$

$$= \lim_{x \to -3} \frac{-2x-3-3}{x+3} = \lim_{x \to -3} \frac{-2x-6}{x+3} = \lim_{x \to -3} \frac{-2(x+3)}{x+3} = -2$$

13) $\lim_{x \to 2} \frac{(2x+1)^2-25}{x-2}$

$$= \lim_{x \to 2} \frac{(2x+1)^2-25}{x-2} = \lim_{x \to 2} \frac{(2x+1-5)(2x+1+5)}{x-2} = \lim_{x \to 2} \frac{(2x-4)(2x+6)}{x-2}$$

$$\lim_{x \to 2} \frac{2(x-2)(2x+6)}{x-2} = 2(10) = 20$$

14) $\lim_{x \to 0^-} \frac{\sqrt{x^2(4+x^3)}}{x}$ $|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$

$$= \lim_{x \to 0^-} \frac{\sqrt{x^2(4+x^3)}}{x} = \lim_{x \to 0^-} \frac{\sqrt{x^2}\sqrt{(4+x^3)}}{x} = \lim_{x \to 0^-} \frac{|x|\sqrt{(4+x^3)}}{x}$$

$$\lim_{x \to 0^-} \frac{\overset{-1}{-x}\sqrt{(4+x^3)}}{x} = \lim_{x \to 0^-} -\sqrt{(4+x^3)} = -2$$

15) $\lim_{x \to 4} \frac{x+\sqrt{x}-6}{4-x}$

$$= \lim_{x \to 4} \frac{(\sqrt{x}+3)\overset{-1}{(\sqrt{x}-2)}}{(2-\sqrt{x})(2+\sqrt{x})} == \lim_{x \to 4} \frac{-(\sqrt{x}+3)}{(2+\sqrt{x})} = \frac{-5}{4}$$

16 ) إذا كانت $h(1)=4$ , $\lim_{x\to 1} h(x)=7$ , $\lim_{x\to 1} f(x)=2$ فأوجد

$$\lim_{x\to 1}(3xf^3(x)-h(x))$$

$$\lim_{x\to 1}(3xf^3(x)-h(x))=\lim_{x\to 1} 3xf^3(x)-\lim_{x\to 1} h(x)$$

$$\lim_{x\to 1} 3x\times(\lim_{x\to 1} f(x))^3-\lim_{x\to 1} h(x)=3(2)^3-7=24-7=17$$

17 ) إذا كانت $\lim_{x\to -1} f(x)=3$ , $\lim_{x\to -1}(f^2(x)+2h(x)-\frac{2}{x})=21$ فأوجد

$$\lim_{x\to -1}(h^2(x))$$

$$\lim_{x\to -1}(f^2(x)+2h(x)-\frac{2}{x})=21\Rightarrow \lim_{x\to -1} f^2(x)+2\lim_{x\to -1} h(x)-\lim_{x\to -1}\frac{2}{x}=21$$

$$(3)^2+2\lim_{x\to -1} h(x)+2=21\Rightarrow 2\lim_{x\to -1} h(x)+11=21\Rightarrow 2\lim_{x\to -1} h(x)=10$$

$$\lim_{x\to -1} h(x)=5\Rightarrow \lim_{x\to -1} h^2(x)=25$$