Mock-MATH12T2E1

مجلس أبوظبي للتعليم
Abu Dhabi Education Council
Education First التعليم أولاً

Mathematics L3 /90 Minutes

ADEC MATH Thanaweya

تجريبي 1 الرياضيات لثانوية ابوظبي

Trimester Two Mock 1

للفصل الدراسي الثاني

2016/2017

2016/2017

Requirement:

Ruler, pencil, protractor, blue pen, scientific calculator.

المتطلبات

مسطرة، قلم الرصاص، منقلة، قلم حبر ازرق، اله حاسبة.

Read these instructions first:

- Complete the box above with your information.
- Write in **blue** pen.
- The paper consists of 4 questions in 10 pages
- Read each question carefully; attempt every one.
- The **total** marks for each question is in[ ]
- Show appropriate working to arrive at your solutions.
- Diagrams/shapes are not drawn to scale.

إقرأ هذه التعليمات أولاً:

- سجل بياناتك قبل البدء بالإختبار.
- اكتب بالقلم الأزرق.
- تتضمن ورقة الأسئلة 4 اسئلة في 7 صفحات
- إقرأ وأجب عن الأسئلة جميعها بدقة.
- تشير الدرجة التي بالمستطيل [ ] إلى درجة السؤال.
- وضح خطوات الحل للوصول إلى الإجابة.
- الرسومات والأشكال البيانية المعطاة **تقريبية**.

Question One:

This section contains 10 multiple choice questions. Each question is worth **2 marks.** Select **one** correct answer only.

Question No.	1	2	3	4	5	6	7	8	9	10	رقم السؤال
Answer	b	c	d	c	a	b	c	a	b	b	الإجابة

Question Two

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}},\ \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}},\ \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

11) Find the exact value of $\mathbf{\sin 67.5^\circ}$ **without** using a calculator

$$\sin(67.5^\circ) = \sin\left(\frac{135^\circ}{2}\right) = +\sqrt{\frac{1-cos135^\circ}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

[PA 4.4] [...../4]

$$\sin 2x = 2\sin x\cos x,\quad \cos 2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x\,,$$

12) Solve the equation $\mathbf{\sin 2x = -\sqrt{3}\cos x}$ if $0° \leq x < 360°$

$$2\sin x\cos x = -\sqrt{3}\cos x$$

$$2\sin x\cos x + \sqrt{3}\cos x = 0$$

$$\cos x\,(2\sin x + \sqrt{3}) = 0$$

Either $\cos x = 0 \implies x = 90° \text{ or } 270°$

Or $2\sin x + \sqrt{3} = 0 \implies \sin x = -\frac{\sqrt{3}}{2}$

x is in the 3^{rd} Quartile or 4^{th} Quartile

Assuming that θ is a reference angle

$$\sin\theta = |\sin x| = \left|-\frac{\sqrt{3}}{2}\right| = \frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

$$then\ x = 180° + 60° = 240°$$

$$or$$

$$x = 360° - 60° = 300°$$

[PA 4.5] [...../8]

13) a) Verify that $\mathbf{\frac{\cot\theta + \tan\theta}{\csc\theta} = \sec\theta}$

$$\frac{\cot\theta + \tan\theta}{\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta}} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\,.\sin\theta = \frac{1}{\cos\theta} = \sec\theta$$

[PA 4.2] [...../7]

b) Verify that $\mathbf{\cos\left(x + \frac{3\pi}{2}\right) = \sin x}$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$ $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

$$\cos\left(x + \frac{3\pi}{2}\right) = \cos(x).\cos\frac{3\pi}{2} - \sin x\,.\sin\frac{3\pi}{2}$$
$$0 - (\sin x)(-1) = \sin x$$

[PA 4.3] [...../3]

14) Simplify the following the following trigonometric expressions?

a) $\sin(x-y)\cos(y)+\cos(x-y)\sin(y)$

$$\sin(x-y)\cos y + \cos(x-y)\sin y$$
$$= \sin(x-y+y) = \sin x$$

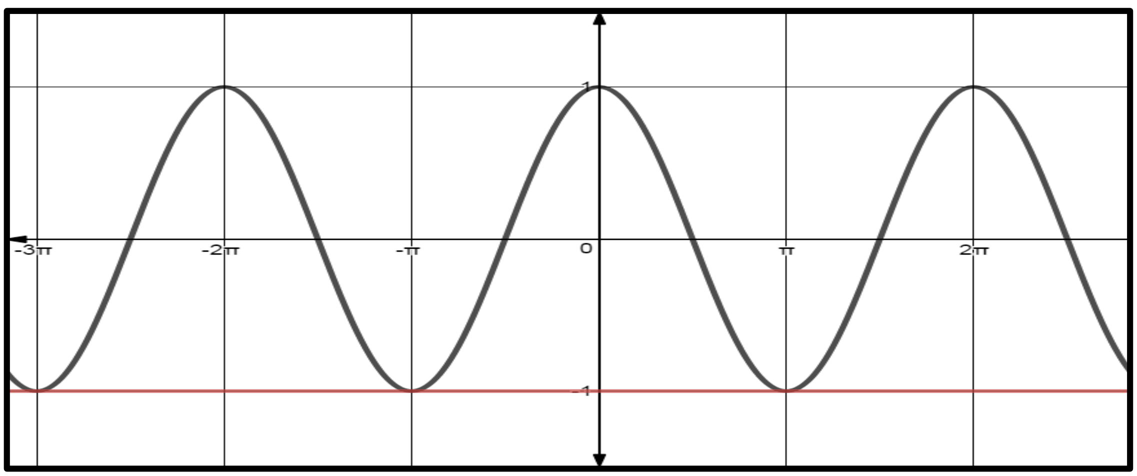
[PA 4.1] [...../2]

b) $\frac{2-4sin^2x}{2\sin 2x}$

$$\frac{2-4sin^2x}{2\sin 2x} = \frac{2\,(1-2sin^2x)}{2(\sin 2x)} = \frac{\cos 2x}{\sin 2x} = cot2x$$

[PA 4.1] [...../4]

15) The graph below represents the equations, $\boldsymbol{y = \cos\theta}$ and $\boldsymbol{y = -1}$. Find **all** the possible solutions when $\boldsymbol{cos\,\theta = -1}$ if $-2\pi \le \theta \le 2\pi$



Answer: $\theta = \pi \ \& \ -\pi$ from the drawing

[PA 4.5] [...../2]

Question Three

16) Hamad kicked a ball from the ground and it reached a maximum ***height*** of ***50 ft.*** The ball returned to the ground level ***40 ft. away*** from the starting point. Assuming that the starting point is at the origin **(0, 0),** Find the equation for the conic section showing the path of the ball in standard form?

Vertex coordinates $(20, 50)$

The equation $(x-h)^2 = 4c\,(y-k)$,

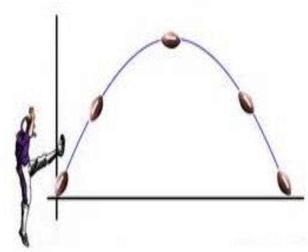
$(x-20)^2 = 4c\,(y-50)$

Choose one of the points $(0, 0)$ or $(40, 0)$

$(0-20)^2 = 4c\,(0-50) \Longrightarrow$

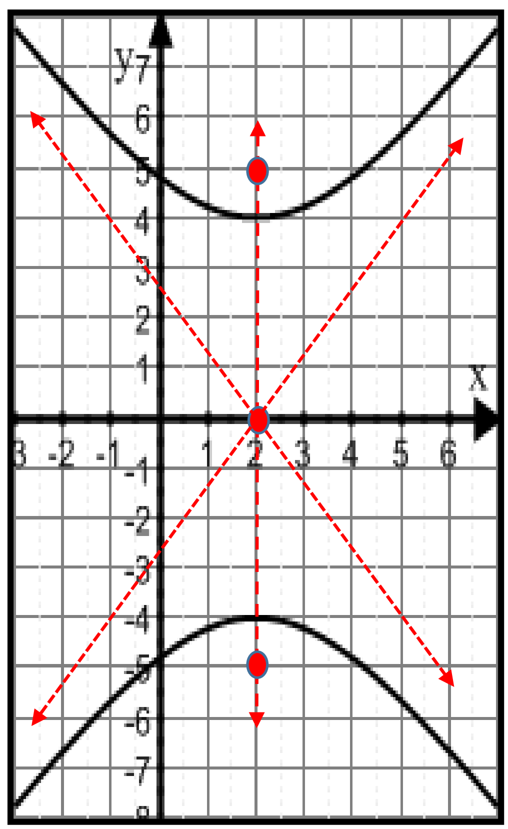
$400 = 4c(-50) \Longrightarrow c = -2$

Then the Equation of the conic is $(x-20)^2 = -8\,(y-50)$



[PA 5.1] [...../5]

17) The equation of one of the asymptotes on the graph below is $3y - 4x + 8 = 0$. Use this **and** the graph to find the following:



a) The name of the conic section of the graph: Hyperbola

b) Identify the **center** of the conic and draw the two axes of symmetry on the graph: (2, 0)

c) $\mathbf{H}(\boldsymbol{x}, \boldsymbol{y})$ Is a point on the curve of the conic. Find the absolute difference for the distance between the point **H** and the foci: From the graph 2a=8

Identify the **foci** and state its coordinates:

$3y = 4x - 8 \Longrightarrow y = \frac{4}{3}(x-2)$

With comparison to the standard form $\Longrightarrow \frac{a}{b} = \frac{4}{3} \Longrightarrow b = 3$

$c = \sqrt{a^2 + b^2} = \sqrt{16+9} = \sqrt{25}$

$c = 5 \Longrightarrow (2, 5)\ \&\ (2, -5)$

d) Draw the two **asymptotes** on the graph.

e) Use the information above to write the **equation** of this conic in **standard form**:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-0)^2}{4^2} - \frac{(x-2)^2}{3^2} = 1$$

$$\frac{y^2}{16} - \frac{(x-2)^2}{9} = 1$$

[PA 5.3] [...../9]

18) A conic section has an equation, $\boldsymbol{y^2 - 4y - 12x + 4 = 0}$, write the following?

a) The equation of the conic in standard form:

$y^2 - 4y = 12x - 4$

$y - 4y + 4 = 12x - 4 + 4 \Longrightarrow (y-2)^2 = 12x$

b) The equation of the directrix:

$4c = 12 \Longrightarrow c = 3$

The vertex $(0,\ 2)$

$x = h - c \Longrightarrow x = 0 - 3 \Longrightarrow x = -3$

[PA 5.1] [...../5]

19) An **ellipse** has its vertices at (-6, -1) and (4, -1) with an **eccentricity** of 0.8. By using this information, find?

a) The **equation** of the ellipse in standard form:

$2a = |4 - (-6)| = 10 \Longrightarrow a = 5$

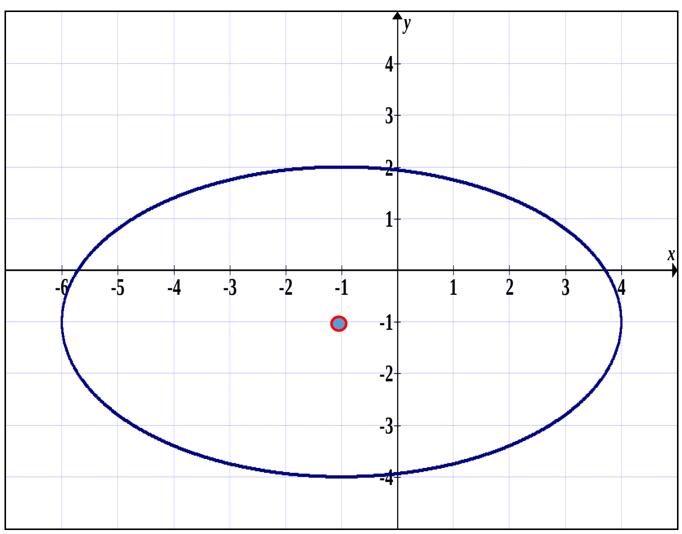
The centre coordinates $\Longrightarrow \left(\frac{4+-6}{2}, -1\right) = (-1, -1)$ (Horizontal)

$\frac{c}{a} = \frac{8}{10} \Longrightarrow \frac{c}{5} = \frac{8}{10} \Longrightarrow c = 4 \Longrightarrow$

$b = \sqrt{a^2 - c^2} = 3$

$\frac{(x+1)^2}{25} + \frac{(y+1)^2}{9} = 1$

b) The sketch of the ellipse



[PA 5.2] [...../8]

Question Four

20)

a) Find the length of the vector $\overrightarrow{AB}$ if it starts at A (5, 2) and ends at B (0, 3)?

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-0)^2 + (2-3)^2} = \sqrt{26}$$

[PA 6.2] [...../3]

b) If **u** = ⟨3, 6⟩ , **v** = ⟨-4, 2⟩ find the product of the vectors **u.v** and verify if they are **perpendicular** or not. Explain your answer.

$$u.v = (3)(-4) + (6)(2) = 0$$

Since u.v = 0 , Then the vectors are perpendicular

[PA 6.3] [...../5]

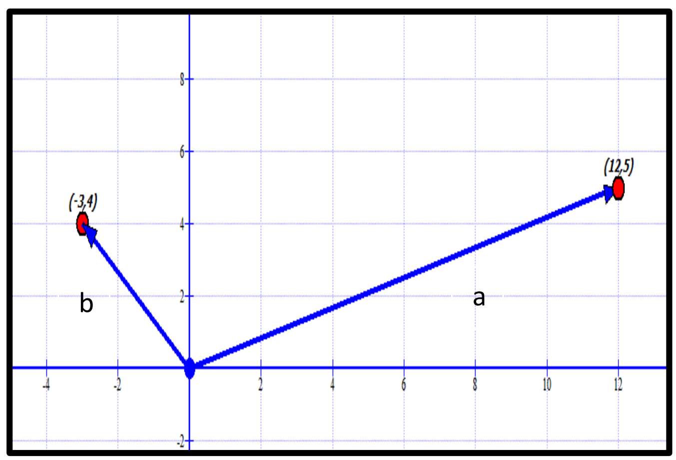
21) Use the graph on the right to find:

a) **a.b**

$$b = \langle -3, 4\rangle \;\&\; a = \langle 12, 5\rangle$$
$$a.b = (12)(-3) + 5(4) = -16$$

b) |**b**|, |**a**|

$$\sqrt{(-3)^2 + (4)^2} = 5$$
$$\sqrt{(12)^2 + (5)^2} = 13$$



c) Find the **angle** between vectors **a** and **b**:

$$\cos\theta = \frac{a.b}{|a|.|b|} = \frac{-16}{(13)(5)} = \frac{-16}{65}$$

$$\theta = cos^{-1}\left(\frac{-16}{65}\right) \approx 104°$$

[PA 6.2, 6.3] [...../9]

22) Abdullah is pulling his daughter Khadijah's cart with a **F**orce of **50N**. He pulls the cart at a distance of **30m** and the **work** is **1060J.**

Find the ***angle*** formed between the force direction and the $\boldsymbol{x}$-axis? (**Ignore the friction force** and you may use $\boldsymbol{w} = \boldsymbol{F} \cdot \overrightarrow{\boldsymbol{AB}}$)

The coordinate of the displacement is $\langle 30, 0 \rangle$

The Horizontal component: $x = F\cos\theta = 50\cos\theta$

The Vertical component: $y = F\sin\theta = 50\,sin\,\theta$

The coordinates of F : $\langle 50\cos\theta, 50\sin\theta \rangle$

$$w = F \cdot \overrightarrow{AB}$$

$$F . \overrightarrow{AB} = \langle F\cos\theta, F\sin\theta \rangle \langle 30, 0 \rangle$$

$$1060 = \langle 50\cos\theta, 50\sin\theta \rangle \langle 30, 0 \rangle$$

$$1060 = 1500\cos\theta + 0$$

$$\cos\theta = \frac{1060}{1500}$$

$$\theta = cos^{-1}\left(\frac{1060}{1500}\right) \approx 45°$$

[PA 6.3] [...../6]

END OF QUESTIONS

(All alternative correct answer should be considered for all questions)